

GUDLAVALLERU ENGINEERING COLLEGE

(An Autonomous Institute with Permanent Affiliation to JNTUK, Kakinada)

Seshadri Rao Knowledge Village, Gudlavalleru – 521 356.

Department of Electrical and Electronics Engineering



HANDOUT

on

Solid State Physics (PH2504)

GUDLAVALLERU ENGINEERING COLLEGE

(An Autonomous Institute with Permanent Affiliation to JNTUK, Kakinada)

Seshadri Rao Knowledge Village, Gudlavalleru – 521 356, Krishna District

Vision of the Institute

To be a leading institution of engineering education and research, preparing students for leadership in their fields in a caring and challenging learning environment.

Mission of the Institute

- To produce quality engineers by providing state-of-the-art engineering education.
- To attract and retain knowledgeable, creative, motivated and highly skilled individuals whose leadership and contributions uphold the college tenets of education, creativity, research and responsible public service.
- To develop faculty and resources to impart and disseminate knowledge and information to students and also to society that will enhance educational level, which in turn, will contribute to social and economic betterment of society.
- To provide an environment that values and encourages knowledge acquisition and academic freedom, making this a preferred institution for knowledge seekers.
- To provide quality assurance.
- To partner and collaborate with industry, government, and R and D institutes to develop new knowledge and sustainable technologies and serve as an engine for facilitating the nation's economic development.
- To impart personality development skills to students that will help them to succeed and lead.
- To instill in students the attitude, values and vision that will prepare them to lead lives of personal integrity and civic responsibility.
- To promote a campus environment that welcomes and makes students of all races, cultures and civilizations feel at home.
- Putting students face to face with industrial, governmental and societal challenges.

Vision

To be a pioneer in electrical and electronics engineering education and research, preparing students for higher levels of intellectual attainment, and making significant contributions to profession and society

Mission

- To impart quality education in electrical and electronics engineering in dynamic learning environment and strive continuously for the interest of stake holders, industry and society.
- To create an environment conducive to student-centered learning and collaborative research.
- To provide students with knowledge, technical skills, and values to excel as engineers and leaders in their profession.

Program Educational Objectives

PEO1: Graduates will have technical knowledge, skills and competence to identify, comprehend and solve problems of industry and society.

PEO2: Graduates learn and adapt themselves to the constantly evolving technology to pursue higher studies and undertake research.

PEO3: Graduates will engage in lifelong learning and work successfully in teams with professional, ethical and administrative acumen to handle critical situations.

HANDOUT ON SOLID STATE PHYSICS

Class & Sem. : I B.Tech – II Semester

Year : 2019-20

Branch : EEE

Credits : 3

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1. Brief History and Scope of the Subject

“Physics is the subject, dealing with Nature & Natural Phenomena”

Every concept that we study in Physics is a consequence of Natural Observation. For example shadow formation led to the concept of Light Nature, etc. A sound knowledge of engineering physics is essential for the engineering student to reach new heights of life.

Physics and Technology

The technological development of any society is very closely related with the applications of physics. Steam engines and the detailed study of heat and thermodynamics were the initiators of the industrial revolution. The development of transistors and development of computers were the initiators of IT revolution. “There is plenty of room at the bottom” the statement made by Feynman started Nano revolution. Today we see the applications of physics in every walk of life. The radio, the television, the wireless, X-rays are used in the identification of fractures in bones. The electricity that we use in our homes and factories is derived from the conversion of some other form of energy great role in technology and in our daily lives. With a basic foundation of physics, you can appreciate and enjoy many more exciting things happening around you.

2. Pre-Requisites

Basic Knowledge of Mathematics, Fundamentals in Physics

3. Course Objectives

- To The course is designed to make the students to learn the conditions for propagation of laser light in guided medium understand principles of solid state materials for use in the engineering applications

4. Course Outcomes:

Upon successful completion of the course, the students will be able to

- explain construction and working of laser
- relate the principles of propagation of light in optical fibers for applications in communications.
- identify conductivity mechanism in semiconductors
- determine types of polarization and classius-mossoti relation
- Differentiate classical and quantum free electron theories
- derive orbital and spin contribution for magnetism

5. Program Outcomes:

The graduates of Electrical and Electronics Communication Engineering program will be able to

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals and an engineering specialization for the solution of complex engineering problems.
2. **Problem analysis:** Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for public health and safety, and cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and Modern engineering and IT tools, including prediction and modeling to complex engineering activities, with an understanding of the limitations.
6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal, and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with the society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning:** Recognizes the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Programme Specific Outcomes (PSO)

1. Apply the knowledge of circuit design, analog & digital electronics to the field of electrical and electronics systems
2. Analyze, design and develop control systems, industrial drives and power systems using modern tools

Mapping of Course Outcomes with Program Outcomes:

MAPPING OF COURSE OUTCOMES WITH PO'S & PSO'S

	1	2	3	4	5	6	7	8	9	10	11	12	PSO1	PSO2
CO1	3			2				1				3		
CO2	3	3		2				1				3		
CO3	3	3		2				1				3		
CO4	2	2						1				3		
CO5	2			2				1				3		
CO6	2			2				1				3		

High-3

Medium-2

Low-1

MAPPING OF COURSE WITH PO'S & PSO'S

SUBJECT NAME	1	2	3	4	5	6	7	8	9	10	11	12	Pso1	Pso2
Solid State Physics	3	2		2	-	-	-	1	-	-	-	3	-	-

7. Prescribed Text Books

1. Engineering Physics by Mani Naidu, Pearson Publications Chennai
2. A text book of Engineering Physics by M.N. Avadhanulu & P.G. Kshirasagar (S. Chand publications)
3. Engineering Physics by Gaur and Gupta.
4. Optics – 5th Edition – Ghatak (TMH Publications)

8. Prescribed Text Books

1. A.J.Dekker, Solid state physics, Published by Macmillan India.
2. Charles Kittel, Introduction to solid state physics, Wiley India Pvt. Ltd.
3. P.K. Palanisamy, Engineering Physics , SciTech publications

9. URLs and Other E-Learning Resources

www.hyperphysics.phy-astr.gsu.edu/HBASE/hframe.html

www.extraphysics.com

www.walter-fendt.de/ph14e

www.phet.colorado.edu

www.sakshat.amrita.ac.in (for virtual lab)

www.forbhongir.blogspot.com

<https://sites.google.com/site/physicsbysureshsaganti>

192.168.0.40/elibrary

www.vlab.co.in

10. Digital Learning Materials:

- <http://nptel.ac.in/courses/>
- <https://onlinecourses.nptel.ac.in>

11. Lecture Schedule / Lesson Plan

S.No	TOPIC	No of. Periods	No of. Tutorials
1.	Unit I LASER		
2.	Introduction	1	2(T)
3.	Characteristics of lasers	1	
4.	Requirement of laser	1	
5.	Spontaneous and Stimulated emission of radiation - Einstein's coefficients	2	
6.	Helium Neon laser.	1	
7.	Semi Conductor laser diode	1	
8.	Co ₂ laser	1	
9.	Problems on unit 1	1	
10.	Unit II Optical fibers		
11.	Construction of Optical fiber	1	2(T)
12.	Principle of Optical Fiber - Total Internal Reflection,	1	
13.	Numerical Aperture and Acceptance Angle, Optical Fibre Construction,	1	
14.	Types of Optical Fibers	1	
15.	Step Index Fibers and Graded Index Fibers	1	
16.	Optical Fiber in communication Block diagram	1	
17.	Advantages of Optical Fibers in Communications.	1	
18.	Problems on unit 2	1	
19.	Unit - III : Semiconductor		
20.	Introduction	1	2(T)
21.	Intrinsic semiconductor - Density of electrons, holes	1	
22.	carrier concentration, Fermi level, conductivity of Intrinsic semiconductor	1	
23.	Extrinsic semiconductor -carrier concentration in conduction band	1	
24.	Drift and diffusion - Einstein's equation	1	
25.	Hall Effect	1	
26.	Direct & indirect band gap semiconductors	1	
27.	Problems on unit 3	1	
28.	Unit - IV : DIELECTRIC PROPERTIES Introduction - Dielectric constant		
29.	Electronic, ionic and orientation polarization	1	2(T)
30.	Estimation of electronic and ionic polarization	2	
31.	internal fields	1	
32.	Clausius - Mossotti equation	1	
33.	Frequency response curve of dielectrics	1	
34.	Dielectric loss, Dielectric Strength, Loss tangent	1	
35.	Problems on unit 4	1	
36.	Unit - V : Free electron and band theory of metals		

37.	Classical free electron theory	1	2(T)
38.	Classical free electron theory – electrical conductivity	2	
39.	Fermi – Dirac (analytical) and its dependence on temperature	1	
40.	Quantum free electron theory	1	
41.	Bloch theorem, Bloch function	1	
42.	Band theory (qualitative) – Kronig – Penney model	2	
43.	Problems on unit 5	1	
44.	Unit VI : Magnetic Materials		
45.	Magnetic permeability – Magnetization	1	2(T)
46.	Origin of magnetic moments	1	
47.	Classification of Magnetic materials – Dia, Para, Ferro	1	
48.	Domain theory (Qualitative)	1	
49.	Hysteresis curve	1	
50.	Soft and hard magnetic materials	1	
51.	Applications	1	
52.	Problems on unit 6	1	
53.	Total	50	

12. Seminar Topics

- classification of magnetic materials
- types of semiconductor

Solid State Physics

UNIT – I

Objective:

The course is designed to make the students

- To comprehend principles of laser.

Syllabus:

Spontaneous and stimulated emission - Einstein's coefficient and their relations - basic characteristics of laser - Basic Requirements of laser - Helium-Neon laser - Semiconductor laser - CO₂ laser - Applications of Laser

Learning Outcomes:

At the end of this chapter students are able to

- summarize the different characteristics of laser
- derive Einstein's coefficients
- analyze construction and working of He- Ne, CO₂, SLD with suitable diagrams
- list uses of lasers

Learning Material

Unit-I

L A S E R S

L1.INTRODUCTION

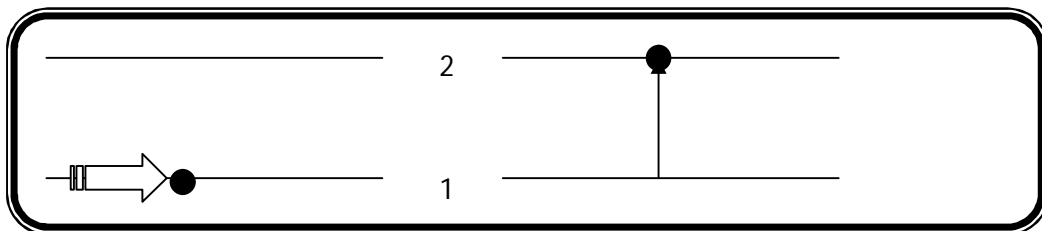
- Laser is an acronym for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation.
- In a laser, the process of stimulated emission is used for amplifying of light waves. Lasers are essentially coherent optical sources.
- It was as early as in 1917 that Einstein first predicted the existence of two different kinds of processes by which an atom can emit radiation. These are called spontaneous and stimulated emission.
- The fact that stimulated emission process could be used in the construction of coherent optical sources was first put forward by Townes and Schawlow in the USA and Bosov and Prokhorov in the USSR. And finally in 1960, Maimann demonstrated the first laser.

L2.BASIC CONCEPTS OF LASER

Absorption, Spontaneous and stimulated emission:

ABSORPTION:

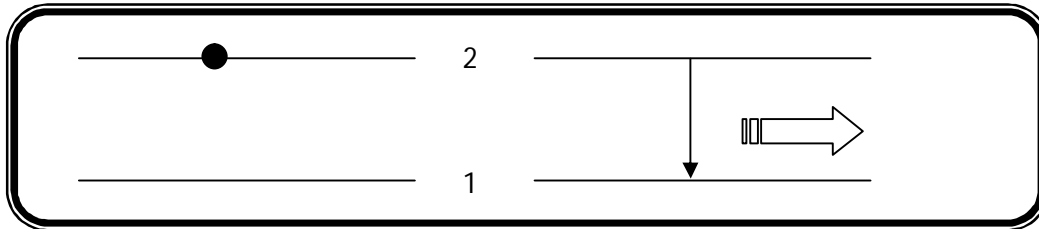
- *The process of particle transfer from normal state corresponding to minimum energy of the system to a higher energy state is termed as excitation and the particle itself is said to be excited. The process is called stimulated or induced absorption.*
- In this process the absorption of energy from the external field takes place.
- The time during which a particle can exist in the ground state is unlimited.
- On the other hand the particle can remain in the excited state for a limited time known as life time (10^{-8} sec).
- The state where the probability of finding the electron is more than excited state is called meta stable.



SPONTANEOUS EMISSION

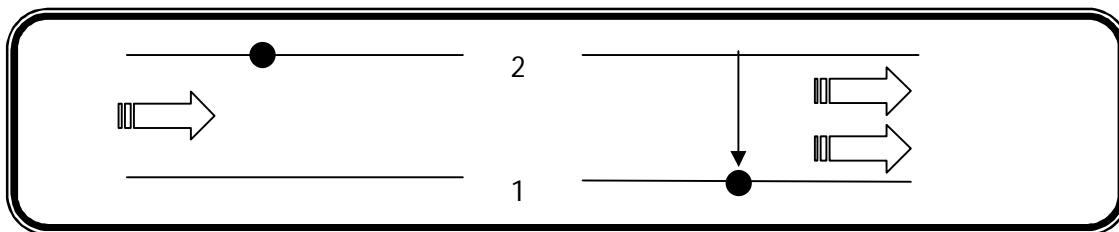
- *The excited atom does not remain in that state for a long time. After short interval of time 10^{-8} Sec it falls to its lower energy state by emitting a photon. Here the excited atom jumps back to its ground state on its own accord and hence the process is known as spontaneous emission.*

- This emission, which takes place without external incitement, is called spontaneous emission.
- The spontaneous emission is random in character. The radiation in this case is a random mixture of quanta having various wavelengths. The waves coincide neither in wavelength nor in phase. Thus the radiation is incoherent and has a broad spectrum.



STIMULATED EMISSION

- ***Electron in the Meta stable state when stimulated by sufficient energetic photon drops to the ground state by emitting another photon which is called as Stimulated emission.***
- Suppose an atom is already in the excited state of energy level E_2 whose ground level energy is E_1 . If at this moment, a photon of energy $h\nu = E_2 - E_1$ is incident on the excited atom. The incident photons stimulate the emission of a similar photon from the excited atom. Now the atom returns to the ground state. The transition takes place much sooner than 10^{-8} sec. The process of speeding up the atomic transition from the excited state to lower state is called stimulated emission.
- The stimulated emission is proportional to the intensity of the incident light.
- The remarkable feature of the stimulated emission is that it is coherent with the stimulating incident radiation. It has the same frequency and phase as the incident radiation.



L3.EINSTEIN COEFFICIENTS

- The probable rate of occurrence of the absorption transition from state 1 to state 2 depends on the properties of states 1 and 2 and is proportional to energy density $u(\nu)$ of the radiation of frequency ν incident on the same. Thus $P_{12} \propto u(\nu)$ or $P_{12} = B_{12}u(\nu)$ --(1)

The proportionality constant B_{12} is known as Einstein's coefficient of absorption of radiation.

- The probability of spontaneous emission from state 2 to state 1 depends only on the properties of states 1 and 2. This is independent of energy density $u(\nu)$ of incident radiation.
- Einstein denoted the probability per unit time by A_{21} .
- (P_{21}) spontaneous = A_{21} . A_{21} is known as Einstein's coefficient of spontaneous emission of radiation.
- The probability of absorption transition depends upon energy density $u(\nu)$ of incident radiation. Whereas the probability of spontaneous emission is independent of it. Hence for equilibrium emission transition depending upon $u(\nu)$ must also exist. Actually these transitions are stimulated emission transitions.

The probability of stimulated emission transition from state 2 to state 1 is proportional to the energy density $u(\nu)$ of the stimulating radiation i.e. (P_{21}) stimulated = $B_{21} u(\nu)$

- Where B_{21} is Einstein's coefficient of stimulated emission of radiation. The total probability for an atom in state 2 to drop to the lower state 1 i.e. therefore $P_{21} = A_{21} + B_{21} u(\nu)$ -----2

RELATION BETWEEN EINSTEIN'S A AND B COEFFICIENTS:-

Consider an assembly of atoms in thermal equilibrium at temperature T with radiation of frequency ν and energy density $u(\nu)$. Let N_1 and N_2 be the number of atoms in energy states 1 and 2 respectively at any instant.

The number of atoms in state 1 that absorb a photon and rise to state 2 per unit time is given by $N_1 P_{12} = N_1 B_{12} u(\nu)$ ----- (3)

The number of photons in state 2 that can cause emission process (spontaneous + stimulated) per unit time is given by $N_2 P_{21} = N_2 [A_{21} + B_{21} u(\nu)]$ ----- (4)

For equilibrium, the absorption and emission must occur equally hence $N_1 P_{12} = N_2 P_{21}$

$N_1 B_{12} u(\nu) = N_2 [A_{21} + B_{21} u(\nu)]$ or $N_1 B_{12} u(\nu) = N_2 A_{21} + N_2 B_{21} u(\nu)$ or $u(\nu) [N_1 B_{12} - N_2 B_{21}] = N_2 A_{21}$

$$\therefore u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} = \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{N_1}{N_2}\right) \left(\frac{B_{12}}{B_{21}}\right) - 1} \text{ ----- (5)}$$

According to Boltzmann distribution law, the number of atoms N_1 and N_2 in energy states E_1 and E_2 in thermal equilibrium at temperature T is given by

$$N_1 = N_0 e^{\frac{-E_1}{kT}} \text{ and}$$

$$N_2 = N_0 e^{\frac{-E_2}{kT}}$$

Where N_0 = Total Number of atoms present and k = Boltzmann's constant.

$$\frac{N_2}{N_1} = \frac{e^{\frac{-E_2}{kT}}}{e^{\frac{-E_1}{kT}}} = e^{\frac{-(E_2-E_1)}{kT}} = e^{\frac{-h\nu}{kT}}$$

Or

$$\frac{N_1}{N_2} = e^{h\nu/kT} \text{-----(6)}$$

Substituting the value of N_1/N_2 from eq. (6) in eq. (5) we get

$$u(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{\left[e^{h\nu/kT} \left(\frac{B_{12}}{B_{21}} \right) - 1 \right]} \text{-----(7)}$$

According to Planck's radiation formula

$$u(\nu) = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{\left[e^{h\nu/kT} \right]} \text{-----(8)}$$

Comparing equations (7) and (8) we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} \text{ and } \frac{B_{12}}{B_{21}} = 1 \text{ or } B_{12} = B_{21}$$

Hence $B_{12} = B_{21}$, the probability of stimulated emission is same as induced absorption

$A_{21}/B_{21} \propto \nu^3$ i.e the ratio of spontaneous emission and stimulated emission is proportional to ν^3 . This shows that the probability of spontaneous emission increases rapidly with energy difference between two states.

L4.CHARACTERISTICS OF THE LASER LIGHT

The most outstanding characteristics of the laser beam are its high degree of directionality, monochromatic, coherence and brightness.

(i) DIRECTIONALITY:- The laser beam is highly directional. To give a simple example a laser beam let us say 10 cm in diameter when beamed at the moon's surface 3,84,000 km away is no more than 5 km wider.

(ii) MONOCHROMATICITY: - A laser produces light in more or less single wave length i.e. the line width associated with laser beams is extremely narrow.

(iii) COHERENCE:- The laser beam is spatially and temporally coherent to an extraordinary degree. It is possible to observe interference effects from two independent laser beams.

(iv) BRIGHTNESS:- The laser beam is highly intense as compared to ordinary sources of light. That is the reason why it can be used for such operations as welding, which involve high temperatures.

L5.BASIC REQUIREMENTS OF LASER

1. Meta stable state
2. Population inversion
3. Pumping
4. Optical resonator

(1) META STABLE STATE:

The state in which possibility of finding electron more is called meta stable state. The lifetime of meta stable state is 10^{-3} sec.

(2) POPULATION INVERSION:

It is the process in which the population of a particular higher energy state is made more than that of a specified lower energy state. Normally the population of ground state is more than the excited state at equilibrium. Let us consider the number of atoms N per unit volume that exist in a given energy state E . This number called population N is given by Boltzmann's equation $N = N_0 e^{-E/kt}$. Here N_0 is the population in the ground state ($E=0$). K_b is the Boltzmann's constant and T is the absolute temperature.

It is clear from the above equation that population is maximum in the ground state and decreases exponentially as one goes to higher energy state. If N_1 and N_2 are the populations in two states, a lower state E_1 and a higher state E_2 we have

$$\frac{N_2}{N_1} = \frac{e^{\frac{-E_2}{kBT}}}{e^{\frac{-E_1}{kBT}}} \text{ from which it follows } N_2 = N_1 e^{-(E_2-E_1)/kBT} \text{ clearly } N_2 < N_1 \text{ since } E_2 > E_1.$$

Since $N_1 > N_2$, whenever an electromagnetic wave is incident, there is net absorption of the radiation. For laser action to take place, it is absolutely necessary that stimulated emission predominate over spontaneous emission. This is, possible only if $N_2 > N_1$. This situation in which $N_2 > N_1$ is called population inversion.

3. PUMPING:

The act of exciting atoms from lower energy state to a higher energy state by supplying energy from an external source is called pumping. The following are the commonly used techniques.

- (a) Optical pumping
- (b) Electric discharges
- (c) Atomic collisions
- (d) Direct conversion and
- (e) Chemical reaction.

OPTICAL RESONATOR: A set of perfect reflector and partial reflector together are known as optical resonator.

L7.He-Ne GAS LASER

- The main drawback of ruby laser is that the output beam is not continuous though very intense.
- For the continuous laser beam gas lasers are used. In gas lasers the vapors of metals are employed as active media.
- The main advantage of gas lasers is exceptionally high monochromatic most pure spectrum and high stability of frequency. Hence they have wide applications in various branches of science and engineering particularly in communications.
- The output power of gas lasers is moderate but inferior to that of crystal lasers. In 1961 A.Javan W.Bennett and D.Herriot reported a continuous gas lasers.

CONSTRUCTION:

Active Medium:

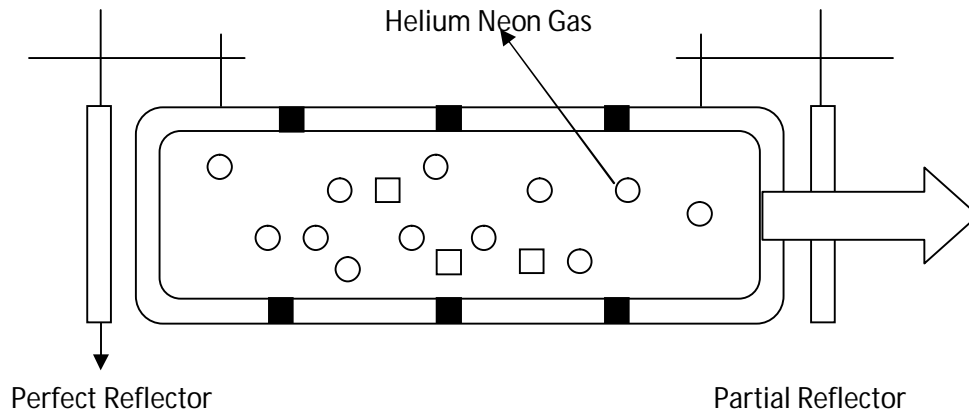
- The gas laser consists of a fused quartz tube with a diameter of about 1.5cm and 80 cm long.
- The tube is filled with a mixture of neon (Ne) under a pressure of 0.1 mm of mercury and Helium under a pressure of 1 mm of mercury.
- There is a majority of helium atoms and minority of neon atoms.

Pumping System:

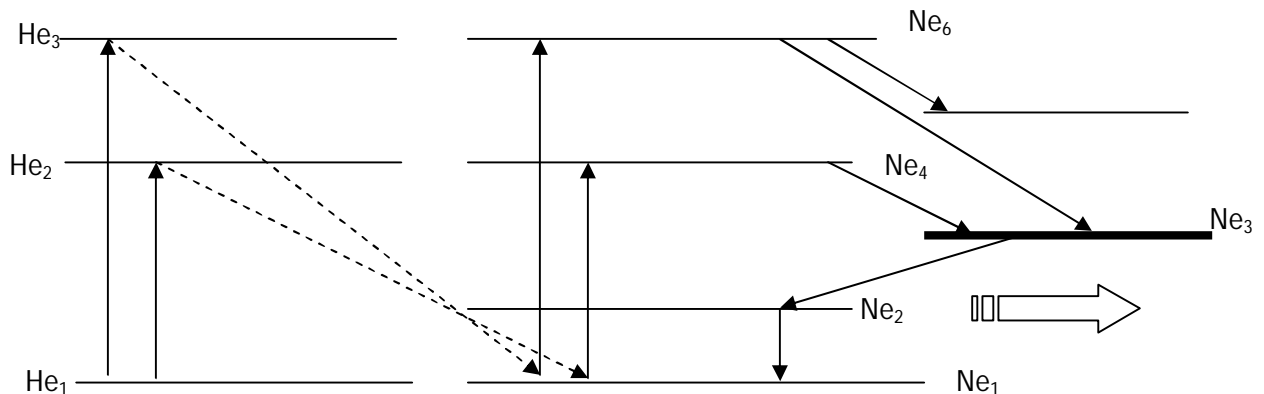
- The pumping system used in He-Ne laser are 1) Electric discharge pumping
2) Inelastic atom-atom collision pumping
- When large potential difference is applied between electrodes anode discharges electrons
- Discharged electrons accelerates towards cathode
- There is a large probability of collision with majority gaseous atoms (He atom)
($e + He \rightarrow He_2^*$)($e + He \rightarrow He_3^*$)
- While excited Helium atoms de excite there is a large probability of colliding with minority gaseous atoms(Ne) in elastically

Optical cavity

- Optical cavity is a set of mirrors kept parallel to each other
- At one end of the tube, there is a perfect reflector while on the other end is a partial reflector.



Working:-

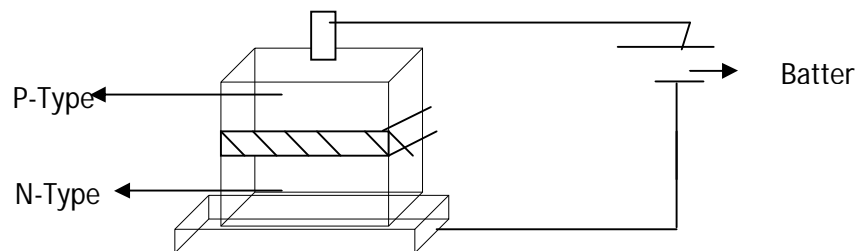


- When a discharge is passed through the gas, the electrons are accelerated towards the positive electrode. During their passage they collide with helium atoms and excite them into the upper states labeled He_2 and He_3 . These are metastable states.
- These atoms remain in these levels for a sufficiently long time. Now these atoms interact with neon atoms, which are in the ground state.
- The interaction excites the neon atoms to their metastable states labeled Ne_4 and Ne_6 . While the helium atoms return to their ground states.
- As the energy exchange continues the population of neon atoms in the excited states increases more and more. When an excited Ne atom passes spontaneously from metastable states Ne_4 and Ne_6 to states Ne_3 and Ne_5 . It emits a photon. This photon travels through the gas mixture.
- If this photon is moving parallel to the axis of the tube, it is reflected back and forth by the reflection unit it stimulates an excited Ne atom and causes it to emit a fresh photon in phase with stimulating photon.
- This process is continued and a laser beam builds up in the tube. When the beam becomes sufficiently intense a portion of it escapes through partially reflecting end.

- The neon atom now passes down spontaneously to a lower state Ne_2 by emitting incoherent light. Finally the Ne-atom comes to ground state through collision with the tube walls. This transition is radiation less.
- The gas lasers are found to emit light, which is more directional and monochromatic. Gas lasers are capable of operating continuously without need of cooling.

L8.SEMI CONDUCTOR LASER

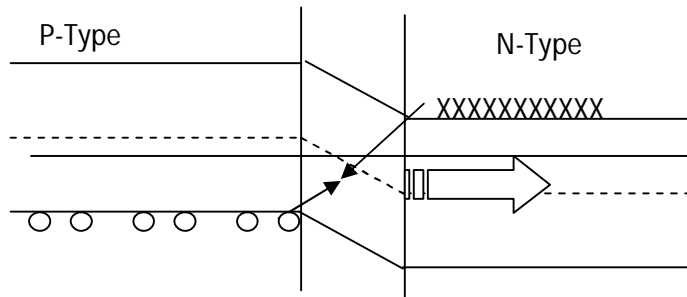
- We know that when a current is passed through a p-n junction. P- region being positively biased holes are injected from P-region into n- region and electrons from n-region into the p-region.
- The electrons and holes recombine and release of energy takes place in or very near the junction region. The amount of this energy called the activation energy or energy gap, depends on the particular type of semiconductor.
- In case of some semiconductors like germanium and silicon, most of the energy is released in the form of heat because the recombination of carriers of opposite sign takes place through interaction with the atoms of the crystals. But in case of other semiconductors such as Gallium Arsenide (GaAs) and others, the energy is released as light because the atoms of the crystals are not involved in the release of energy.
- The wavelength of emitted light depends on activation energy of the crystal. Photons emitted at the moment of recombination of an electron with a hole will stimulation recombination of other carriers of electric charges. The result will be stimulated emission of radiation.
- If these radiations moving in the plane of the junction are made to move back and forth in the plane of the junction by reflection at opposite parallel sides and perpendicular to the plane of junction, a very powerful laser beam of stimulated radiation can be produced.



- In the first models of semiconductor laser, the active medium was a single crystal of gallium arsenide GaAs cut into a plate having a thickness of 0.5 mm.
- The platelet consists of two parts exhibiting electron conductivity and a hole conductivity respectively. The emission is stimulated in p-n junction layer, which of course is very thin. Electric current is applied to the crystal platelet through a strip electrode fixed to its upper surface.
- When exciting currents are small only a small part of carriers undergo recombination the process is spontaneous. The laser radiation is random and

incoherent. But when the current density is increased the emission becomes more and more coherent and the radiation intensity markedly increases.

- Now efficiency of GaAs lasers reaches 40 per cent. When cooled to 20°C semiconductor lasers have delivered an output of more than two watt of continuous power.



L9.CO₂ LASER

- The carbon dioxide laser invented in the year 1963 by Kumar Patel. Carbon dioxide lasers are the highest power continuous wave lasers that are currently available.
- The CO₂ laser produces a beam of infrared light with the principal wavelength bands centering around 9.4 and 10.6 micrometer.

- **Construction**

Any laser is made up of three main parts
Active Medium, Pumping, Optical Cavity

- **Active Medium:**

The active medium is CO₂ gas. As, He is used for excitation of Ne atoms in the He-Ne laser, in CO₂ laser for efficient excitation of CO₂ molecules, N₂ molecules are used. Addition of He to the gas mixture enhances the efficiency. The ratio of pressures of CO₂:Ne:He is 1:4:5. The tube is about 2.5cm in dia & 5m in length.

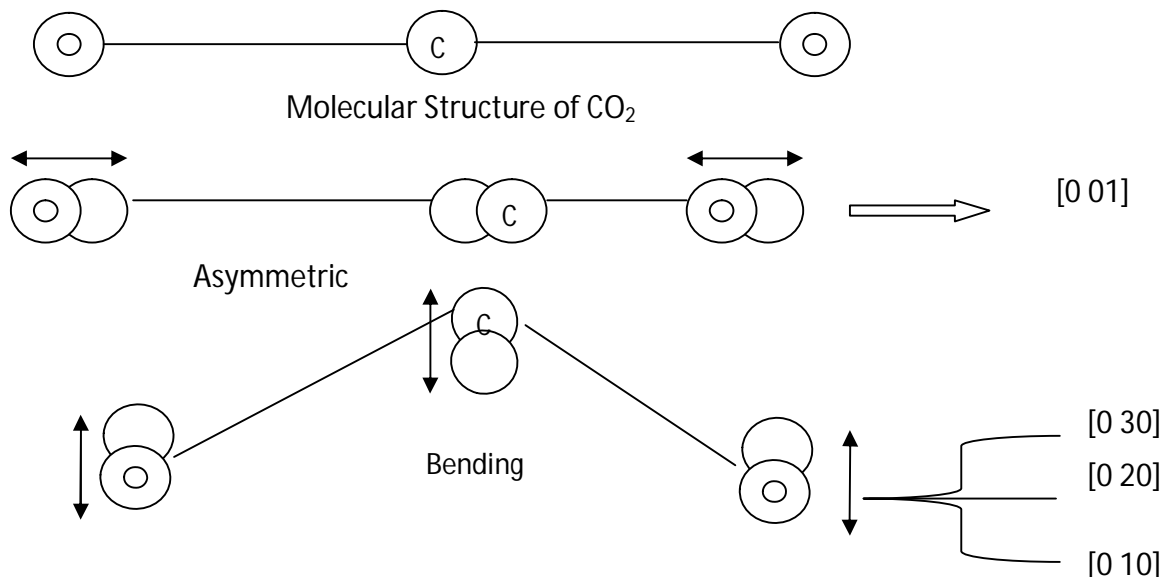
- **Pumping:**

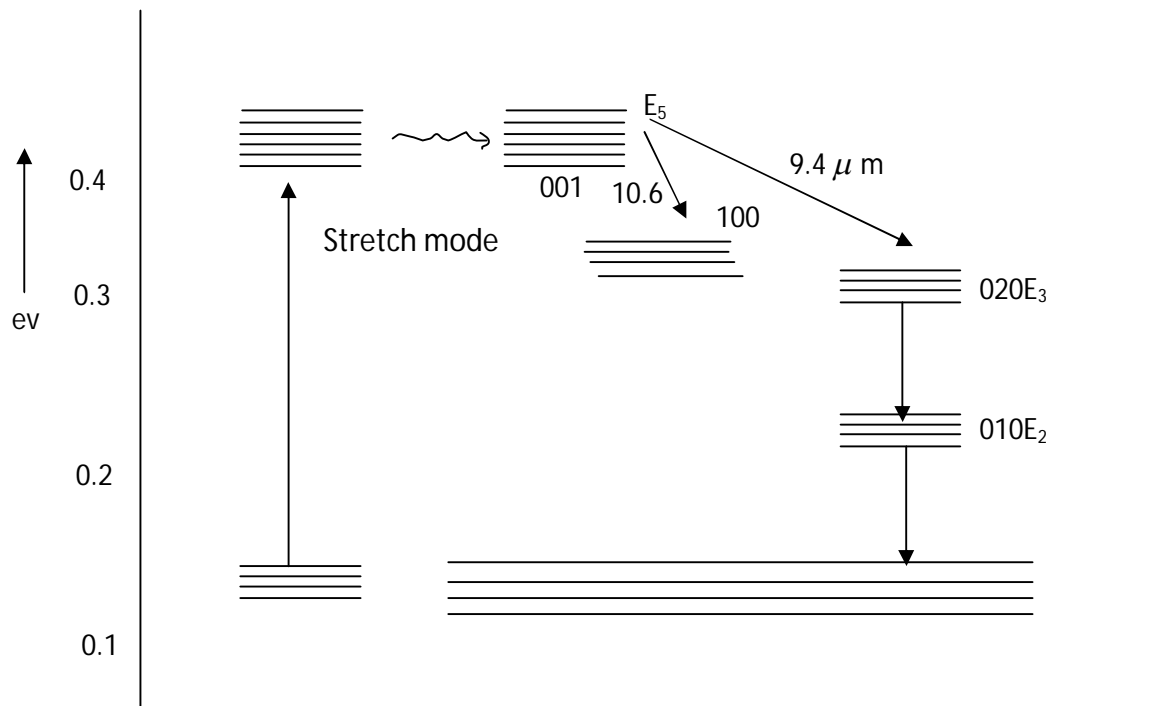
In CO₂ laser the excitation is produced by electric discharge excited N₂ molecules transfer energy to the CO₂ molecules in resonant collisions.

Optical Cavity:

Because CO₂ lasers operate in the infrared region, special materials are necessary for their construction. The mirrors are made of coated silicon, molybdenum or gold while windows and lenses are made of either germanium (or) Zinc Selenide.

Vibration energy levels in CO₂ Molecule.





Laser Action in CO₂ Laser:

1. When a discharge is passed through tube the nitrogen molecules are excited and are raised to higher excited state.
2. The excited energy of nitrogen molecules is transferred to carbon dioxide molecules through collisions and carbon dioxide molecules are raised to their excited vibrations energy level E₅ [0 01] from their ground state.
3. The energy level E₅ is a meta stable state energy level. Hence there is a population inversion.
4. Stimulating photons of wavelength 10.6 μm and 9.6 μm induce CO₂ molecules to undergo stimulated emission by laser transitions E₅ to E₄ giving laser of 10.6 μm & from E₅ to E₃ 9.6 μm.
5. CO₂ molecules from E₄ and E₃ are returned to their ground through fast decay and diffusion.

L9.APPLICATIONS OF LASERS

LASERS have wide applications in different branches of science

- I. Due to the narrow bandwidth lasers are used in microwave communication.
- II. Due to narrow angular spread, the laser beams have become means of communication between earth and moon or other satellites. The earth moon distance has been measured with the use of lasers.
- III. By the use of lasers the storage capacity of information in computers is greatly improved.
- IV. In industries lasers are used in welding, drilling and cutting. They can blast holes in diamond and hard steels.

- V. They have been used successfully in the treatment of detached retinas. Microsurgery is also possible because laser beams can be focused on very small areas.
- VI. Their study is also oriented for military purposes. They can serve as a war weapon it has been predicted that they become legendary 'death ray'.
- VII. Chemical applications:
They can initiate or hasten certain chemical reactions, which could not possible in the absence of suitable photons.

Solid State Physics

UNIT-I

Assignment-Cum-Tutorial Questions on LASER

Section : A (*Objective Questions*)

- The phase of the incoherent radiation changes with _____.
(a) time (b) amplitude (c) velocity (d) None of the above
- When the matter is interacting with radiation _____ should take place.
- According to the Einstein's theory, the probability of induced absorption process is same as that of _____.
- Under population inversion condition, the number of atoms in the higher energy state is _____ than the lower energy state in an atomic system.
- The wavelength of He-Ne laser beam is _____.
- The ratio of He atoms to Ne atoms in the He-Ne laser system is _____.
- Define lifetime?
- What is meant by population inversion?
- Distinguish between spontaneous and stimulated emissions
- Write any four applications of lasers?

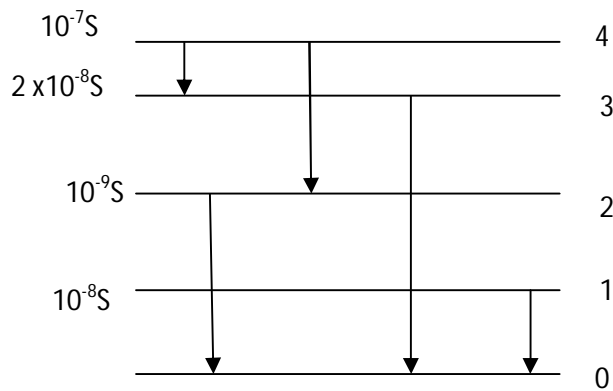
Section B: (*Subjective Questions*)

- What is meant by a laser? Explain the characteristics of lasers. (CO1)
- With the help of suitable diagrams explain the principle, construction and working of He-Ne gas laser. (CO1)
- What is population inversion? Explain how it is achieved in He-Ne laser. (CO1)
- Derive the relation between the probabilities of spontaneous emission and stimulated emission in terms of Einstein Coefficients. (CO1)
- What are the various applications of lasers? (CO1)
- Describe the construction and working of a homo junction semiconductor laser and what are the advantages of hetero junction over homo junction laser (CO1)
- Explain the construction and working of CO₂ laser. (CO1)

Section C: (*Additional Questions*)

- In Laser beam, the photons are in phase, the resultant intensity is given in terms of amplitude 'a' and number of photons 'n' is proportional to ()
(A) na (B) n²a (C) n²a² (D) na²
- Calculate the Wavelength of radiation emitted by an LED made up of a semiconducting material with band gap energy 2.8eV. ()
(A) 44 m (B) 44.30 x 10⁻⁸ cm (C) 4830 x 10⁻⁸ cm (D) 499 x 10⁻⁸ cm
- For InP laser diode, the wavelength of light emissions 1.55 μm. What is its band gap in eV? ()
(A) 8014 eV (B) 8.014 eV (C) 0.8014eV (D) 1480 eV
- Calculate the long wavelength limit of an extrinsic semiconductor if the ionization energy is 0.02 eV.
- A carbon-dioxide laser has a nominal wavelength of 10.6 micrometers. What is its frequency? ()
(A) 3,00,000 hertz (B) 2.8 x 10¹³ hertz

- (C) 1.06 gigahertz (D) 2.8×10^{10} hertz
6. How do two light waves of the same wavelength and amplitude interfere if they are 180 degrees out of phase? ()
 (A) Destructively (B) constructively
 (C) Partially, producing a wave with amplitude $\sqrt{2}$ times the two input waves (D) None of the above
7. Why would an engineer select a red He-Ne laser over a red semiconductor laser? ()
 (A) The He-Ne is smaller (B) The He-Ne uses power more efficiently
 (C) The He-Ne has better beam quality (D) The He-Ne costs less
8. A semiconductor has a band gap of 1.5 electron volts. At what wavelength will it emit light if it can operate as a laser? **(827nm)**
9. Five energy levels of a system including the ground state are shown below. Their lifetimes and the allowed electric dipole transitions are also marked. **(GATE 2009)**



- (A) $1 \rightarrow 0$ (B) $2 \rightarrow 0$ (C) $4 \rightarrow 2$ (D) $4 \rightarrow 3$
10. Which of the following technologies/devices uses LASER
 (A) CD writer (B) Barcode Scanner (C) LASIK (D) All of the above
[NPTEL-FUNDAMENTALS OF LASER]
11. In a 3 level system stimulated emission is normally triggered by
 (A) Absorption (B) Pumping source
 (C) Thermal decay (D) Spontaneous emission
[NPTEL-FUNDAMENTALS OF LASER]
12. In a He-Ne laser partial pressure of He and Ne are respectively
 (A) 1 mbar and 0.1 mbar (B) 1 mbar and 0.5 mbar
 (C) 2 mbar and 0.03 mbar (D) 1 mbar and 0.01 mbar
[NPTEL-FUNDAMENTALS OF LASER]
13. Which one of the following electronic transitions in neon is NOT responsible for LASER action in a helium-neon laser? **(GATE 2006)**
 (A) $6s \rightarrow 5p$ (B) $5s \rightarrow 4p$ (C) $5s \rightarrow 3p$ (D) $4s \rightarrow 3p$
14. In He-Ne Laser, the laser transition takes place in **(GATE 2007)**
 A) He only B) Ne Only C) Ne first, than in He D) He first, than in Ne
15. The coherence length of laser light is **(GATE 2008)**
 (A) directly proportional to the length of the active lasing medium.
 (B) directly proportional to the width of the spectral line.
 (C) inversely proportional to the width of the spectral line.

- (D) Inversely proportional to the length of the active lasing medium
16. The population inversion in a two level laser material CANNOT be achieved by optical pumping because (GATE 2009)
- (A) the rate of upward transitions is equal to the rate of downward transitions
- (B) the upward transitions are forbidden but downward transitions are allowed
- (C) the upward transitions are allowed but downward transitions are forbidden
- (D) the spontaneous decay rate of the higher level is very low
17. Einstein's coefficients for absorption and stimulated emission, B_{12} and B_{21} are related by
- (A) $B_{12} = B_{21}$ (B) $B_{12} > B_{21}$ (C) $B_{12} < B_{21}$ (D) None of the above
- [NPTEL-FUNDAMENTALS OF LASER]
18. Which of the following is a unique property of LASER
- (A) Mono chromaticity (B) Coherence (C) Collimated (D) All of the above
- [NPTEL-FUNDAMENTALS OF LASER]
19. Calculate the wavelength of emission from GaAs material whose energy band gap $E_g = 1.44$ eV. (Planck's constant = 6.625×10^{-34} J-s)
20. A matter wave propagating with a velocity 3×10^3 m/s and its wavelength 600nm. Calculate matter wave energy.
21. Energy gap of a semiconductor 3 eV. Calculate wave length of emitted photons.
22. GaAs has a band gap energy of 1.43 eV at 300K. Determine the wavelength above which an intrinsic photo detector fabricated from this material will cease to operate.
23. A laser beam has a wavelength of 8×10^7 m and aperture 5×10^{-3} m. The laser beam is sent to the moon. The distance between moon and earth is 4×10^5 km. Calculate the angular spread of the beam.

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Solid State Physics

UNIT - II FIBER OPTICS

Objective:

The course is designed to make the students

- to learn the conditions for propagation of laser light in guided medium.

Syllabus:

Basic principle of optical fiber - Construction of optical fiber - Acceptance angle, Acceptance cone - Numerical Aperture - Types of optical fiber - Light wave communication by using optical fiber

Learning Outcomes:

At the end of this chapter students are able to

- estimate the expression for numerical aperture
- categorize types of optical fibers
- device optical fiber for communication

Solid State Physics
UNIT - II
FIBER OPTICS

O1. INTRODUCTION

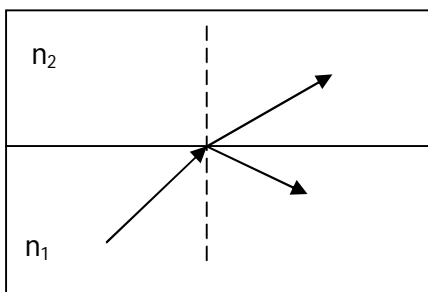
- The transmission of light along transparent cylinders by multiple total internal reflections is a fairly old and well-known phenomenon. **John Tyndall** at the Royal Society in England gave the earliest recorded scientific demonstration of this phenomenon in 1870.
- The interest in this phenomenon described by Tyndall was dormant until 1927. When the possibility of using fibers for transmitting television pictures was tried out. However these ideas were not pursued. A new burst of activity began in the early 1950's when the transmission of pictures along an aligned bundle of flexible glass fibers was carried out with remarkable success. This led to the development of 'flexible fiberscope'.
- Thus the subject of fiber optics was established with the advent of lasers, the subject of fiber optics received an added impetus. Currently throughout the world fiber optics are flexible hair thin threads of glass or plastic.

O2. BASIC PRINCIPLE: TOTAL INTERNAL REFLECTION

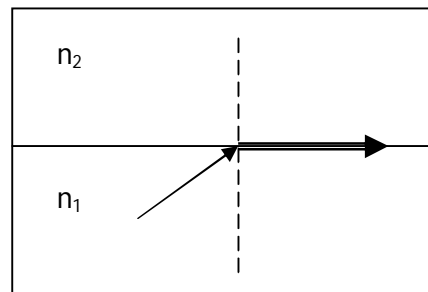
- For the phenomenon of TIR to take place the two conditions are to be satisfied
 1. Light should travel from denser medium to rarer medium.
 2. The angle of incidence should be greater than the critical angle θ_c of the medium.
- **Critical angle** θ_c of a medium is the value of the incident angle at which the angle of refraction is 90° .

Consider n_1 is the refractive index of denser medium and n_2 is the refractive index of rarer medium.

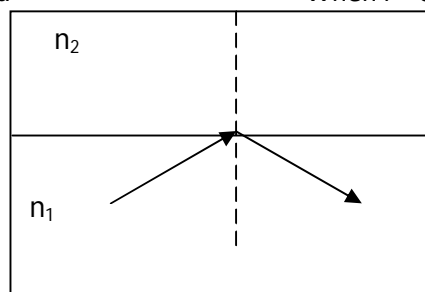
When a light ray traveling in a medium of refractive index n_1 strikes a second medium of refractive index n_2 the angle of refraction θ is given by Snell's law $n_1 \sin i = n_2 \sin \theta$ when a ray of light traveling from a optically dense medium to a less dense medium;



When $i < \theta_c$ it is refracted



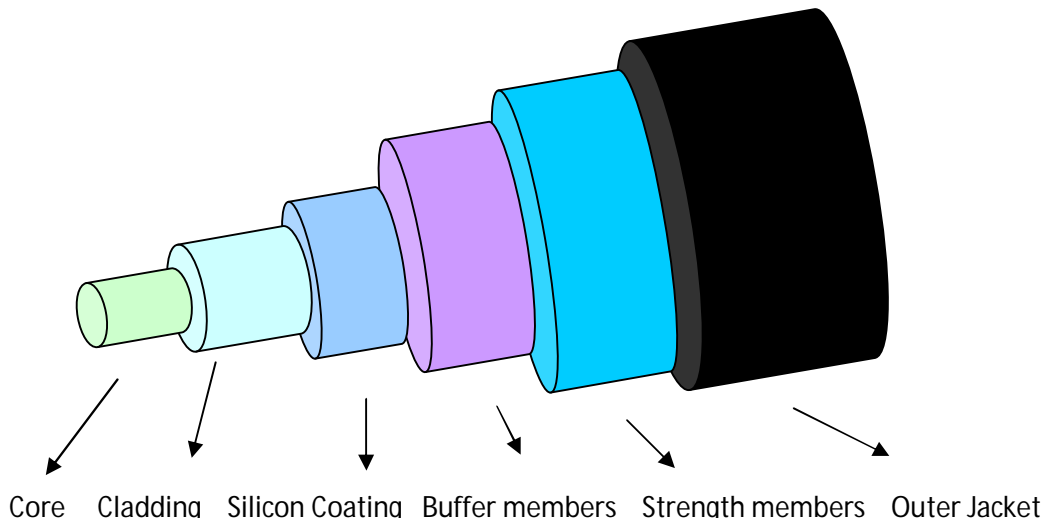
When $i = \theta_c$ it is traverses along the



It will not be refracted if it strikes the surface at an angle equal to or greater than a particular angle called critical angle.

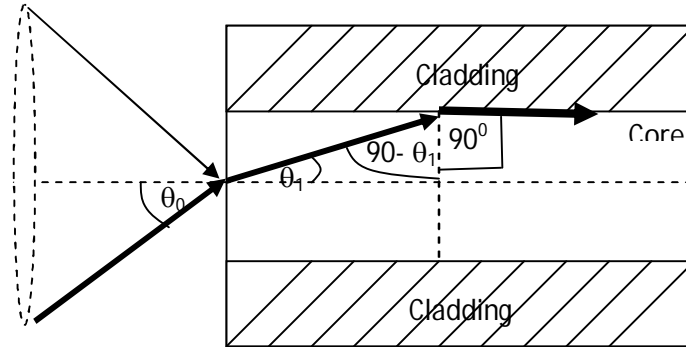
- Instead it will be totally reflected at the surface between the two media such a phenomenon occurs because when a ray of light travels from D.M. to L.D.M the refracted ray bends away from the normal.
- Consider the case $\theta = 90^\circ$ $n_1 \sin \theta_c = n_2 \sin 90^\circ$ $\sin \theta_c = \frac{n_2}{n_1}$. Thus for any ray whose angle of incidence is greater than this θ_c . Total internal Reflection occurs.

O3.Fibre Construction



- It consists of a central cylinder or core surrounded by a layer of material called the cladding, which in turn is covered by a jacket.
- Light is transmitted within the core the cladding keeps the light waves within the core.
- The cladding is having less refractive index protects the core. The cladding also provides some strength to the core. The additional jacket protects the fiber from moisture and abrasion.
- The core as well as the cladding is made of either glass or plastic with these materials three major types of fibers are made.
 1. Plastic core with plastic cladding
 2. Glass core with plastic cladding
 3. Glass core with glass cladding.
- The core is generally made of Polystyrene or Poly methyl meta acrylate where as the cladding is generally made of silicone or Teflon.
- The glass is made of silica small amounts of components such as boron, germanium or phosphorous is added to change refractive index of the fiber core
- Core diameters range from 5 to 600 μm .
- Cladding diameters range from 125 to 750 μm .
- Optical fiber diameter is of the order of 0.1 mm.

O4 Numerical Aperture And Acceptance Angle



- Let n_0 , n_1 and n_2 be the refractive indices of surrounding medium, core of the fiber, and cladding respectively.
- Now for refraction at the point of entry of the ray from air into the core, we have by applying the Snell's law that $n_0 \sin \theta_0 = n_1 \sin \theta_1$ ----- (1).
- At the point core-cladding of the surface, the angle of incidence = $90 - \theta_1$ and angle of refraction is 90° .
- Again applying Snell's law, we have $n_1 \sin(90 - \theta_1) = n_2 \sin 90$ or $n_1 \cos \theta_1 = n_2$ or $\cos \theta_1 = \frac{n_2}{n_1}$ ----- (2).

Rewriting equation (1) we have

$$\sin \theta_0 = \frac{n_1}{n_0} \sqrt{1 - \frac{n_2^2}{n_1^2}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} . \text{ If the medium surrounding the fiber is air then}$$

$$n_0 = 1 \text{ or } \sin \theta_0 = \sqrt{n_1^2 - n_2^2} \text{ if } NA = \sqrt{n_1^2 - n_2^2} .$$

- If θ_1 is the angle of incidence of an incident ray, then the ray will be able to propagate if $\theta_1 < \theta_0$ or if $\sin \theta_1 < \sin \theta_0$ or $\sin \theta_1 < \sqrt{n_1^2 - n_2^2}$

This is the condition for propagation i.e., $\sin \theta_1 < NA$

- **Fractional Index change:**

The fractional index change Δ is the ratio of refractive index difference between the core and cladding to the refractive index of core of an optical fiber.

$$\therefore \Delta = \frac{n_1 - n_2}{n_1} \text{ ----- (3)}$$

Relation between N.A. and Δ

$$\text{From Equation (3) } (n_1 - n_2) = n_1 \Delta \text{ ----- (4)}$$

$$\text{We have } NA = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{(n_1 + n_2)(n_1 - n_2)} = \sqrt{(n_1 + n_2)n_1\Delta}$$

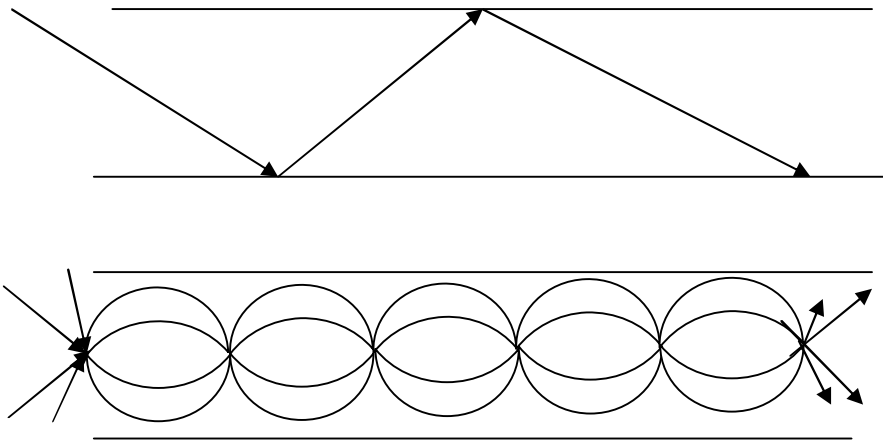
since $n_1 \approx n_2$ ($n_1 + n_2$) = $2n_1$.

$$NA = \sqrt{2n_1^2\Delta} \quad \text{or} \quad N.A = n_1\sqrt{2\Delta}$$

CHECK YOUR PROGRESS: Obtain the expression for Numerical Aperture of an Optical fiber?

O5. Step Index Fiber and Graded Index Fiber

- Optical fibers guide light by either reflection or refraction depending on the types of the fiber used.
- In the reflective types light travels in a zigzag fashion.
- In the refractive type light rays travel in a continuous curve. In either case light rays are confined to the core.

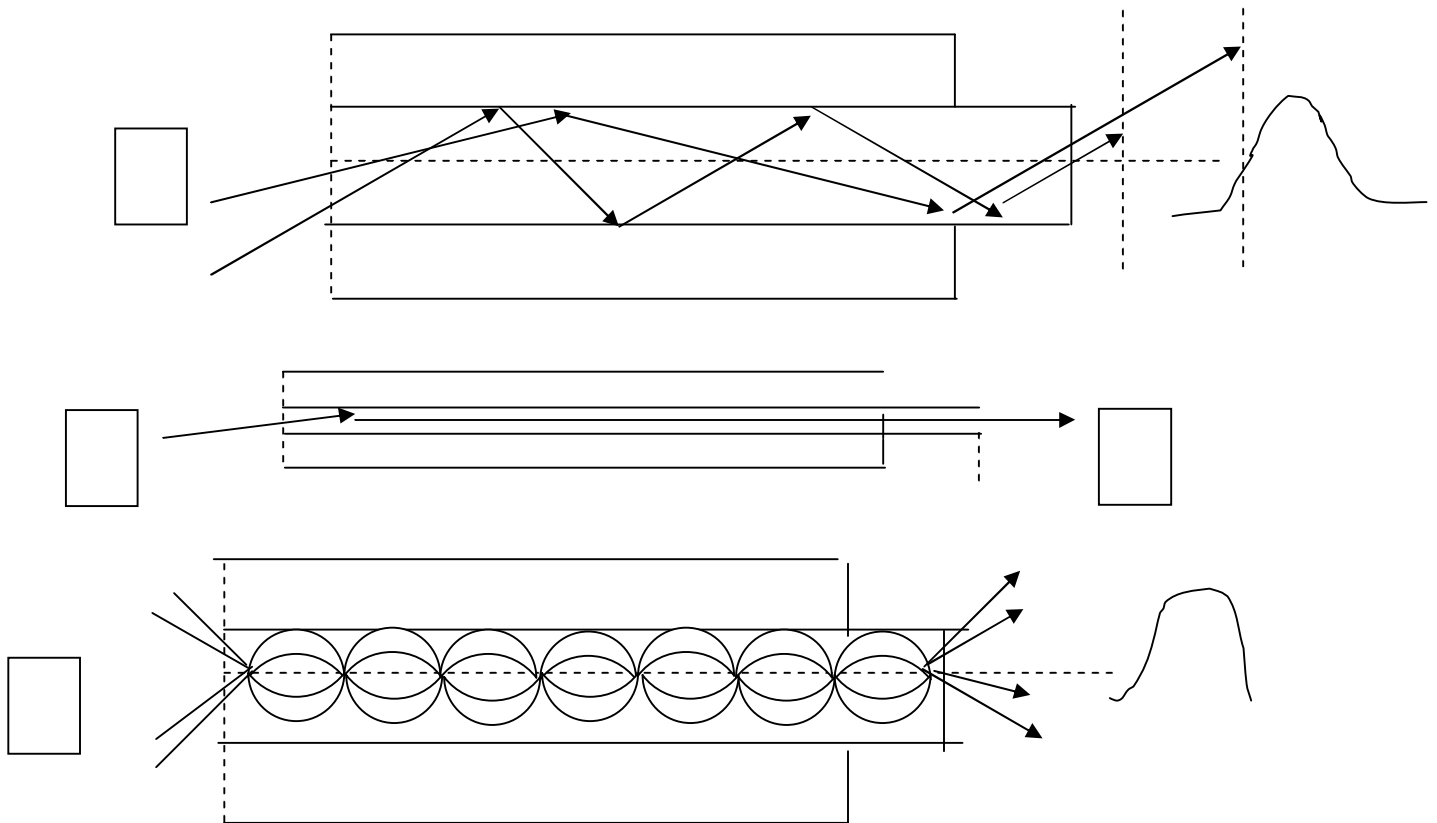


- There are two kinds of reflective type's optical fibers
 1. Single mode step index fiber.
 2. Multi mode step index fiber.
- There is only one type of refractive fiber called the multimode graded index.
- Mode means simply the various paths light can take in a fiber multimode means that several paths are available.
- The distinction between a step index and graded index fiber is as follows:
In optical fibers $n_1 > n_2$. In a step index fiber the refractive index is constant throughout the core. In such a fiber the refractive index profile abruptly changes at the junction of the core and the cladding because of the abrupt change they are called step index optical fiber.
- In a graded index optical fiber refractive index varies continuously across the core. It is highest of the centre of the core and tapers off rapidly towards the outer edge.

- **DISTORTION IN OPTICAL FIBRE**

When light travels in an optical fiber, each ray of light may be reflected hundreds or thousands of times. The rays reflected at high angles the higher order modes must travel a greater distance than the low angle rays to reach the end of the fiber. Because of this longer distance, the higher angle rays arrive later than the

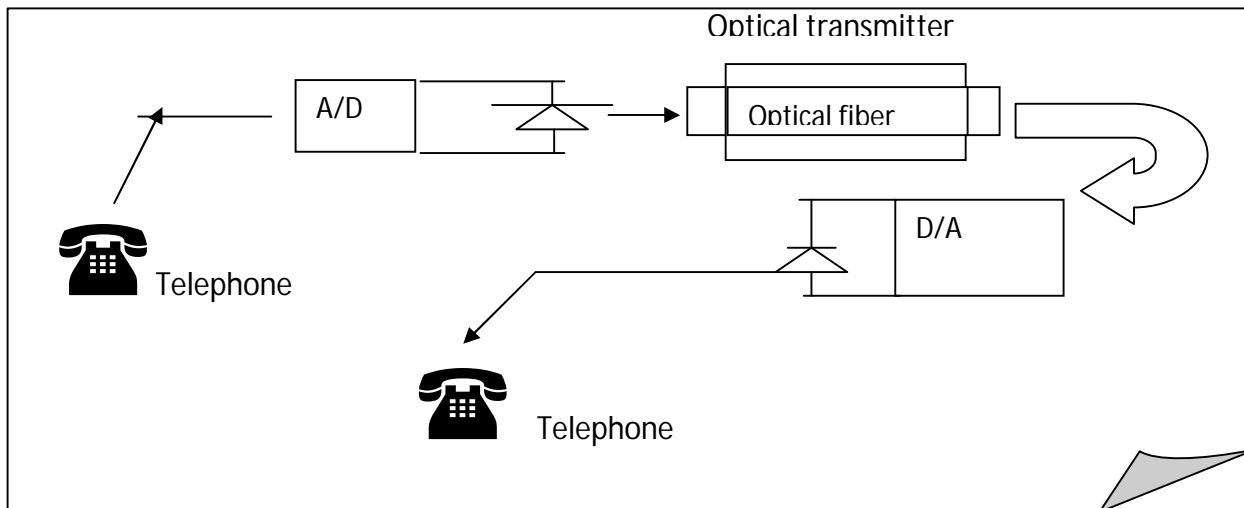
lower angle rays. As a result modulated light pulses broaden as they travel down the fiber. The output pulses then no longer exactly match the input pulses causing signal distortion.



- In a graded index fiber, light rays travel at different speeds in different parts of the fiber because the refractive index varies throughout the fiber near the outer edge the refractive index is lower. As a result rays near the outer edge travel faster than rays in the centre at the core. Because of this all the rays arrive at the end of the fibre at approximately, the same time.
- In effect light rays in these fibers are continuously refocused as they travel down the fiber. This in effect reduces signal distortion.
- In single mode step index fiber also, the signal distortion is reduced. But the problem is one of aligning different fibers.
- Another source of signal distortion is chromatic dispersion. When the light is not monochromatic, different wavelengths have different velocities in glass. Therefore all portions of the input light pulse will not arrive simultaneously at the output. This distortion is called chromatic dispersion.

CHECK YOUR PROGRESS: Explain signal distortion in optical fibers?

06. LIGHT WAVE COMMUNICATION BY USING OPTICAL FIBER



Optical Receiver

- (1) Subscriber's telephone (where sound waves get converted into corresponding electrical signals).
- (2) Wire pairs carrying electrical voice signals
- (3) Encoder (where electrical continuous signals are coded into digital electric pulses by means of analogue to digital converters)
- (4) Optical transmitter containing amplifier and LED/Semiconductor laser, which generate optical pulses as it is driven by the output from the encoder.
- (5) Optical fibre which transmits the optical pulses
- (6) Optical receiver containing a photodiode and an amplifier whose outputs are electrical pulse coded signals.
- (7) Decoder (where digital signals are converted back into continuous electrical signals, by means of digital to analogue converters).
- (8) Wire pairs, which carry the electrical, output signals from the decoder.
- (9) Subscriber's telephone (where electrical signals get converted into corresponding sound waves).

CHECK YOUR PROGRESS: Explain light wave communication by using Optical Fiber?

07 Advantages of Optical Fiber

- (1) Extremely wide band width
- (2) Smaller diameter, lighter-weight cables
- (3) Lack of cross talk between parallel fibers.
- (4) Immunity to inductive interference.
- (5) Potential of delivering signals at low cost
- (6) Must faster than copper cables
- (7) Longer life span
- (8) Temperature resistant
- (9) Easy maintenance

Applications

The technology of fiber optics is intimately linked with its applications in medicine, communication system, Sensors and industry. The initial efforts in fiber optics were concerned with the development of endoscope for the visualization of internal parts of human body. In industry, it could be used to examine welds, nozzles and combustion chambers inside, jet aircraft engines, which would be inaccessible for observation otherwise, etc.,

Solid State Physics
UNIT - II
Assignment-Cum-Tutorial Questions on optical fiber

Section A: Objective Questions

1. Define Numerical Aperture.
2. The expression for numerical aperture of an optical fiber is_____
3. In multimode graded index fiber, light rays travel _____in different parts of the fibre.
4. Total internal reflection takes place when the angle of incidence is _____the critical angle.
5. Optical fibers are made up with _____materials.
6. Optical fibers carry very large information compared to copper cables because of _____
7. In the structure of fiber, the light is guided through the core due to total internal _____
a. reflection b. refraction c. diffraction d. dispersion
8. Which type of fiber optic cable is most widely used?
a) Single mode step index b) Multimode step-index
c) Single-mode graded-index d) Multimode graded

Section B: (Subjective Questions)

1. Explain the principle behind the functioning of an optical fiber.(CO2)
2. What is meant by acceptance angle for an optical fiber? Obtain mathematical expression for acceptance angle and numerical aperture. (CO2)
3. Explain difference between the step index fiber and graded index fiber. (CO2)
4. With neat ray diagram explain the propagation of light in an optical fiber by block diagram (CO2)
5. What are the various types of optical fibers explain. (CO2)
6. Explain Light wave communication by using optical fiber. (CO2)
7. Distinguish between optical fiber and copper wave communication. (CO2)
8. What are the various applications of optical fibers? (CO2)

Section C : (Additional Questions)

1. In a fiber-optic cable, what determines the maximum speed of the data pulses the cable can handle? ()
A) Absorption B) attenuation C) Bandwidth D) cladding
2. Which of the following does not cause losses in optical fiber cables? ()
[IES-2009]
a) Impurities b) Microbending c) Attenuation in glass d) Stepped index operation
3. Calculate the numerical aperture of an optical fiber whose core and cladding are made of materials of refractive index 1.6 and 1.5 respectively. ()
A) 0.86 B) 0.55 C) 0.78 D) 0.98

4. On which bands, do the optical fibers operate? [IES-2009] ()
 1. Ultra Violet band 2. Ultra High Frequency band 3. Visible light band
 4. Infra red band
 Select the correct answer from the codes given below:
 a) 1 only b) 1 and 2 only c) 1,2 and 3 d) 1,3 and 4
5. Total internal reflection takes place when light travels from ()
 A) Rarer to denser medium B) Any medium to any different medium
 C) Air to liquid (or) Solid D) A denser to a lighter medium
6. Consider the following advantages of optical fiber-cables
 1. Small diameter
 2. Immunity to cross talk and electromagnetic interference
 3. Laser and LED modulation methods lend themselves ideally to digital
 Operation ()
 Which of these advantages are correct? [IES-2011]
 a) 1 and 2 only b) 2 and 3 only c) 3 and 1 only d) 1,2 and 3
7. Why are semiconductor lasers important for communication engineering?
 1. They Possess large band width 2. They operate at low voltage 3.They are
 suitable for coupling t fibers at wavelengths, where the attenuation is very
 small 4. Their integration with other components is easy
 Select the correct answer using the code given below: [IES-2008]
 a) 1 only b) 1 and 2 only c) 2 and 3 only d) 1,2,3, and 4
8. Light from free space is incident at an angle θ_i to the normal of the facet of
 a step-index large core optical fibre. The core and cladding refractive
 indices are $\theta_1= 1.5$ and $\theta_2= 1.4$, respectively. The maximum value of θ_i
 (in degrees) for which the incident light will be guided in the core of the
 fibre is _____ [GATE-Electronics-2016]
9. In an optical fibre, the pulse dispersion effect is minimized by
 [IES-Electronics-2016]
 i) Using a high frequency light source ii) Using plastic cladding
 iii) Minimizing the core diameter
 which of the above statement is/are correct?
 a) i only b) ii only c) iii only d) i, ii and iii
10. In an optical fiber, the concept of Numerical aperture is applicable in
 describing the ability of __ [IES-2008]
 a. Light Collection b. Light Scattering
 c. Light Dispersion d. Light Polarization
11. In the structure of a fiber, which component provides additional strength
 and prevents the fiber from any damage? [IES-2008]
 a. Core b. Cladding c. Buffer Coating d. None of the above
12. Which of the following is not a common application of fiber-optic cable?
 [IES-2008]
 a) Computer networks b) Long-distance telephone systems
 c) Closed circuit TV d) Consumer TV
13. The **cladding** which surrounds the optic fiber line is used
 [ISRO 2015- Optical fiber [EE]]

- (a) To protect the fiber
 - (b) To reduce optical interference
 - (c) Because it help to guide light in the core
 - (d) To ensure that refractive index remains constant
14. Optical fiber operates on the principle of [NPTEL]
- (a) Total internal reflectance
 - (b) Tyndall effect
 - (c) Photo-electric effect
 - (d) Laser technology
15. Calculate the acceptance angle and the numerical aperture of a given optical fiber, if the refractive indices of core and cladding are 1.563 and 1.498 respectively.
16. If an optical fiber has refractive indices of core and cladding as 1.53 and 1.42 respectively, then calculate its critical angle.
17. A light ray enters core of refractive index 1.55 through the end face from a medium of refractive index 1.6 with an angle of incidence 60° . Calculate its angle of refraction θ_1 at the interface.
18. If the fractional difference between the core and cladding refractive indices of fiber is 0.0135 and numerical aperture NA is 0.2425, calculate the refractive index of core and cladding materials.

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SOLID STATE PHYSICS

Unit – III- PHYSICS OF SEMI CONDUCTORS

Objective:

- Identify the appropriate semi conductor materials for engineering applications.

Syllabus:

Properties of Fermi Dirac energy distribution function - Concentration of carriers in conduction band, valance band - Intrinsic carrier concentration - Drift and diffusion currents - Einstein's relations - Hall Effect - Applications of Hall Effect

Learning Outcomes:

At the end of the unit students are able to

- Value the significance of Fermi distribution function.
- Compute carrier concentrations in energy bands.
- Interpret the behavior of semiconductors in external magnetic field – Hall effect
- Differentiate Drift, Diffusion Currents and relate them through Einstein coefficients.
- Distinguish between elemental and compound semiconductors.

S0. Properties of Fermi Dirac energy distribution function

The Fermi-Dirac probability distribution of energies is given by $P_e(E) =$

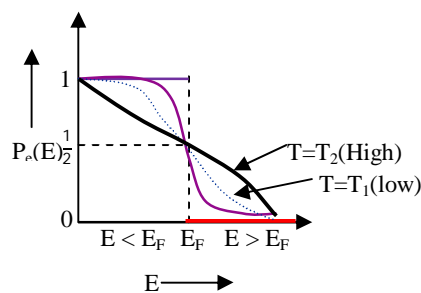
$$\frac{1}{1 + \exp^{(E-E_F)/KT}}$$

Properties of the Fermi Dirac statistics:-

1. It is derived on the condition that equilibrium exists and is therefore strictly valid only in equilibrium.
2. E_f denotes an energy level and is called the Fermi level. From the above it also follows that the Fermi level concept is valid only. Strictly speaking at equilibrium.
3. The Fermi-Dirac distribution applies to all particles obeying Pauli's exclusion principle and is equally applicable regardless of the type of solid (Insulator, Semiconductor, metal) doping of semiconductor etc.
4. The Fermi-Dirac distribution considers statistically the entire collection of fermions in the volume. Thus it considers all electrons in the semi conducting solid and not merely electrons in a band.
5. Whenever an electron state is empty it can be characterized as a hole. The Fermi-Dirac distribution function for holes in the solid would correspond to the statistical distribution of vacant states. Denoting the hole distribution function as $P_h(E)$. We obtain $P_h(E) = 1 - P_{FD}(E)$.

$$P_h(E) = \frac{1}{1 + \exp^{(E_F - E)/KT}}$$

6. At an energy $E = E_F$ the probability of occupancy of the electron state by an electron or hole is $\frac{1}{2}$. This provides us with a definition of the Fermi level.
7. At OK $P_e(E) = 1$ for $E < E_F$ and $P_e(E) = 0$ for $E > E_F$ This implies that at absolute zero temperature all states up to the Fermi level are full i.e., occupied by electrons and all states above the Fermi level are empty.
8. The distribution function is a strong function of temperatures only at energies close to E_F . plots of $P_e(E)$ at different temperatures are as follows.



9. The Fermi-Dirac distribution gives the probability of an electron state of energy "E" being occupied and thus represents the fraction of available states being occupied. The actual number of occupied states would be given by the number of states available times the proportion of some occupied. The number of available states per unit volume are given by $g_e(E)$, $g_h(E)$ for electrons and holes respectively. The number density n of electrons and P of holes at energy E would be given by

$$n(E) = g_e(E)P_e(E)$$

$$P(E) = g_h(E)P_h(E)$$

10. An extension of the principle can be used to obtain carrier densities n_0 and P_0 in a range or band of energies from E_1 to E_2 in equilibrium.

$$n_0 = \int_{E_1}^{E_2} g_e(E) dE p_e(E)$$

$$P_0 = \int_{E_1}^{E_2} g_h(E) dE p_h(E)$$

• **Electron and Hole densities:-**

The most important application of Fermi-Dirac statistics in semiconductors is for computing free electron and hole densities.

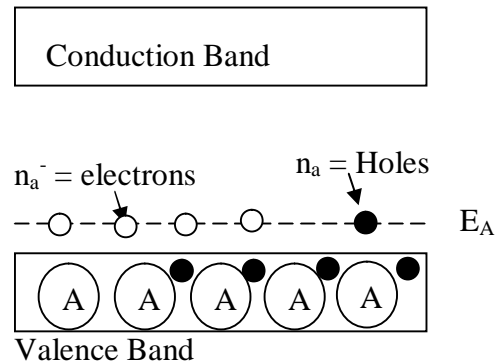
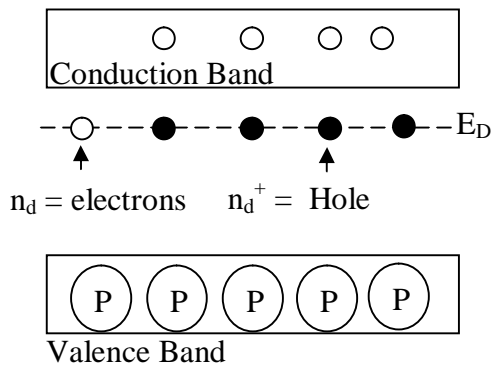
- Consider an extrinsic n-type semiconductor. The impurity of donor level E_d lies very close to the conduction band in the energy gap. Let N_D be the number of impurity atoms per unit volume. Each of the impurity atoms gives rise to a single electron state at E_d .
- If E_f is the Fermi energy level at a temperature T . The number of electrons at the donor level E_d is given as $n_d = g_e(E_d) P_e(E_d)$ where $g_e(E_d)$ is the number of available states per unit volume $P_e(E)$ is the Fermi function, Hence

$$n_d = \frac{N_D}{1 + \exp\left(\frac{E_d - E_f}{KT}\right)} \text{-----(1)}$$

Where n_d represents the un-ionized donor atoms out of the available total N_D states per unit volume. Thus the remaining vacant states are to be treated as n_d^+ ionized donors per unit volume is given by

$$n_d^+ = N_D - n_d = \frac{N_D}{1 + \exp\left(\frac{E_f - E_d}{KT}\right)} \text{-----(2)}$$

Where each of the n_d^+ levels being empty is associated with a +ive charge that corresponds to the absence of an electron.

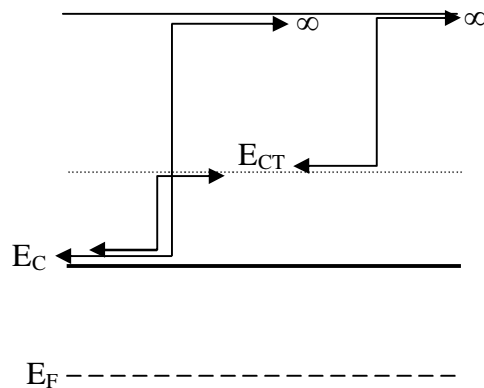


- For semiconductors doped with N_A acceptor atoms per unit volume that give rise to acceptor level E_a that is a little above the valance energy E_v . If some of the N_A atoms get ionized by accepting an electron then the number of acceptor atoms n_a^- occupying E_a level is given by

$$n_a = \frac{N_A}{1 + \exp\left(\frac{E - E_F}{KT}\right)} \quad \text{-----(3)}$$

Then the number of unionized acceptor atoms n_a will be = $\frac{N_A}{1 + \exp\left(\frac{E_F - E}{KT}\right)}$ Hence the acceptor levels also can be either filled or empty.

S1. Concentration of carriers in conduction band



The two types of free carriers of interest in a semiconductor are the electrons in the conduction band and holes in the valance band. Let n be the number of electrons per unit volume of a homogeneously doped semiconductor crystal in equilibrium. If the conduction band extends from E_C to E_{CT} then the number of electrons n per unit volume in the conduction band is

$$n = \int_{E_C}^{E_{CT}} g_e(E) dE P_e(E)$$

$$n = \int_{E_C}^{\infty} g_e(E) dE P_e(E) - \int_{E_{CT}}^{\infty} g_e(E) dE P_e(E) = I_1 - I_2$$

The Fermi level lies always below E_C and for reasonable widths of conduction band $E_{CT} - E_F \gg KT$. In I_2 , all energies E being considered are greater that E_{CT} and can be written as $E_{CT} + \Delta E$ where ΔE is positive.

$$I_2 = \int_{E_{CT}}^{\infty} \frac{g_e(E) dE}{1 + \left[\exp\left(\frac{E_{CT} - E_F}{KT}\right)\right] \exp^{\Delta E/KT}}$$

Since all terms are positive and the second term in the denominator approaches infinity the integral I_2 zero. Hence substituting $g_e(E) dE$ in I_1 and setting I_2 to zero we write the electron carrier concentration n in the conduction band.

Density of states: Let $g(E) dE$ represents the number density of available energy states lying between E and dE . Naturally for energies lying in the forbidden bands there are no available states and so in these ranges of energy $g(E) dE = 0$.

- Consider next an allowed band extending from E_c to E_{ct} . To find the number density of electron states in this allowed band in a crystal we will utilize Heisenberg's uncertainty principle.
- In one dimensional space the minimum uncertainty is given by $\Delta x \cdot \Delta P_x = h$ and in three dimensional space.

$$\Delta x \Delta y \Delta z \Delta P_x \Delta P_y \Delta P_z = h^3 \text{-----(1)}$$

If we consider a volume V of a crystal the uncertainty in momentum as ΔT_P then $V \Delta T_P = h^3$. $\Delta T_P = h^3/V$ -----(2)

- In this momentum space we can allow two electron states of opposite spin as dictated by Pauli's exclusion principle. The location of an electron in these states is not specified it is only known to be within the boundaries of the crystal hence the associated particle momentum P varies continuously from zero. The energy of the electron state near the minima E_c of band is given by

$$E = E_c + \frac{P^2}{2m_e} \text{-----(3)}$$

Further in a range of momentum from 0 to p in a three dimensional real crystal the number of distinct momenta is the set of distinct points contained within a radius of p and corresponds to the volume of the sphere. $\frac{4}{3}\pi p^3$.

- The number T_p of momenta states within range p and $p+dp$ are obtained by differentiating $T_p dp = 4\pi p^2 dp$
- In the minimum volume momenta space ΔT_p , there are two electron states in a momentum volume h^3/V since in the momentum range defined by p & $p+dp$ there are $4\pi p^2 dp$ momenta states the number of electron states in this volume will be

$$G(p)dp = \frac{4\pi p^2 dp^2}{h^3/V} = \frac{8\pi p^2 dp v}{h^3}$$

- The density of states per unit volume $g(p)dp = \frac{G(p)dp}{v} = \frac{8\pi p^2 dp}{h^3}$ -----(4)

For our purpose it is convenient to express the density of states as a function of energy in a conduction band utilizing equ(3)

$$P^2 = 2m_e(E-E_c) \text{---(5)} \quad P = \sqrt{2m_e} (E - E_c)^{1/2}$$

$dp = \frac{1}{2} \sqrt{2m_e} (E - E_c)^{-1/2} dE$ -----(6) substituting equ(5) & (6) in (4) we obtain effective density of states as a function of energy.

$$g_e(E)dE = \frac{8\pi 2m_e(E - E_c) (2m_e)^{1/3} (E - E_c)^{-1/2}}{h^3 \cdot 2} dE$$

$$g_e(E)dE = \frac{1}{2\pi^2} \left(\frac{2m_e}{h^2} \right)^{3/2} (E - E_c)^{1/2} dE$$

$$n = \int_{E_c}^{\infty} g_e(E) dE p_e(E) = \int_{E_c}^{\infty} \frac{1}{2\pi^2} \left(\frac{2me}{h^2} \right)^{3/2} \frac{(E - E_c)^{1/2}}{1 + \exp \frac{E - E_F}{kT}} dE$$

In this integral the energy E being considered always greater than E_c if the temp T is such that $E_c - E_F \geq 5KT$ we can ignore 1 in the denominator. Let $x = \frac{E - E_c}{KT}$ in the integral. The limits changes as the lower limit $E_c \rightarrow 0$ while the upper limit as remains unaltered.

$$(E - E_c)^{1/2} = x^{1/2} (KT)^{1/2} dE = KT dx$$

$$\exp \left(\frac{E - E_F}{KT} \right) = \exp \left(\frac{E - E_c}{KT} \right) \exp \left(\frac{E_c - E_F}{KT} \right) = e^x e \left(\frac{E_c - E_F}{KT} \right)$$

Hence using above things we get

$$n = \frac{1}{2\pi^2} \left(\frac{2me}{h^2} \right)^{3/2} \int_0^{\infty} \frac{(KT)^{1/2} x^{1/2} e^{-x} (KT) dx}{\exp \left(\frac{E_c - E_F}{KT} \right)}$$

$$\frac{1}{2\pi^2} \left[\frac{2meKT}{h^2} \right]^{3/2} \exp \left(\frac{E_F - E_c}{KT} \right) \int_0^{\infty} x^{1/2} e^{-x} dx$$

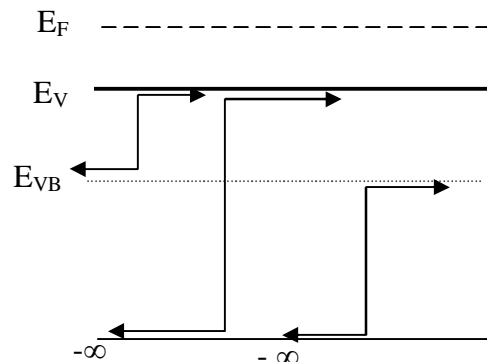
But the standard integral $\int x^{1/2} e^{-x} dx = \frac{\sqrt{\pi}}{2}$

$$n = \frac{1}{4} \left(\frac{2meKT}{h^2 \pi} \right)^{3/2} \exp \left(\frac{E_F - E_c}{KT} \right)$$

$$= 2 \left(\frac{2\pi meKT}{h^2} \right)^{3/2} \exp \frac{E_F - E_C}{KT}$$

$N = N_e \exp \frac{E_F - E_c}{KT}$ where $N_e = 2 \left(\frac{2\pi meKT}{h^2} \right)^{3/2}$ which is known as the effective density of states in the conduction band.

S2. Concentration of carriers in valance band



- The hole density P in a homogeneous doped semiconductor is obtained by

$$P = \int_{E_{vb}}^{E_v} g_h(E) dE Ph(E) = \int_{-\infty}^{E_v} g_h(E) dE Ph(E) - \int_{-\infty}^{E_{vb}} g_h(E) dE Ph(E) = I_1 - I_2$$

As before $I_2 \rightarrow 0$ substituting for $g_h(E)dE$ and $Ph(E)$ in I_1 we have

$$P = \int_{-\infty}^{E_v} \frac{1}{2\pi^2} \left(\frac{2mh}{h^2} \right)^{3/2} \frac{(E_v - E)^{1/2} dE}{1 + \exp \frac{E_F - E}{KT}}$$

again for temp $E_F - E_v$

$$P = \int_{-\infty}^{E_v} \frac{1}{2\pi^2} \left[\frac{2me}{h^2} \right]^{3/2} \exp \frac{E - E_F}{KT} (E_v - E)^{1/2} dE \text{ let } \frac{E_v - E}{KT} = x$$

Hence

$dE = -KTdx$ then $(E_v - E)^{1/2} = (KT)^{1/2} x^{1/2}$ and $\exp \frac{E - E_F}{KT} = \exp \frac{-(E_v - E)}{KT} \exp \frac{E_v - E_F}{KT}$ the limits tends to $0 \rightarrow \infty$

$$P = \frac{1}{2\pi^2} \left(\frac{2mh}{h^2} \right)^{3/2} \exp \left(\frac{E_v - E_F}{KT} \right)$$

$P = N_v \exp \frac{E_v - E_F}{KT}$ where $N_v = 2 \left(\frac{2\pi mhKT}{h^2} \right)^{3/2}$ is the effective density of states in valance band.

S3. Intrinsic carrier concentration

- Absolutely pure semiconductors are called intrinsic semiconductors. The free carriers in an intrinsic semiconductor are generated due to thermal energy as electron-hole pairs. Hence the electron concentration n_i in conduction band will be equal to the concentration of holes p_i in the valance band.
- Let E_i be the Fermi level of the intrinsic semiconductor in equilibrium substituting n_i , p_i and E_i in extrinsic n&p concentrations.

$$n_i = 2 \left(\frac{2\pi meKT}{h^2} \right)^{3/2} \exp \frac{E_i - E_c}{KT} \text{----- (1)}$$

$$p_i = 2 \left(\frac{2\pi mhKT}{h^2} \right)^{3/2} \exp \frac{E_v - E_i}{KT} \text{----- (2)}$$

But $n_i = p_i$ ----- (3) substituting (1) & (2) in (3) gives

$$\exp \left[\frac{E_i - E_c - E_v + E_i}{KT} \right] = \left[\frac{mh}{me} \right]^{3/2}$$

$$\text{Thus } 2E_i - (E_c + E_v) = KT \ln \left(\frac{mh}{me} \right)^{3/2}$$

$$\text{Or } E_i = \frac{E_c + E_v}{2} + \frac{3}{4}KT \ln\left(\frac{m_h}{m_e}\right) \text{-----(4)}$$

If the effective masses $m_e = m_h$ the Fermi level $E_i = \frac{E_c + E_v}{2}$ i.e. E_i lies midway between E_c and E_v which happens to be the centre of the band gap at any temperature. If $m_h > m_e$ E_i linearly increases towards E_c . The material properties E_c , E_v , m_h & m_e determine the value of E_i .

Combining equations (1) (2) & (3) yield

$$n_i^2 = 4\left(\frac{2\pi K}{h}\right)^3 (m_h m_e)^3 T^3 \exp\frac{E_v - E_c}{KT} \text{-----(5)}$$

We define $A = 4\left(\frac{2\pi K}{h}\right)^3 (m_h m_e)^3 T^3$ ----- (6) and

$E_g = E_c - E_v$ ----- (7) hence substituting (6) & (7) in (5) we obtain

$$n_i^2 = AT^3 \exp\frac{-E_g}{KT} \text{-----(8)}$$

Where A and E_g are constants for a given semiconductor and n_i is called the intrinsic concentration which indicates the thermally generated electrons and holes. It is a strong function of the temperature T .

- The equilibrium concentration n and p can be expressed in terms of the characteristic constants n_i are given by

$$n = 2\left(\frac{2\pi m_e KT}{h^2}\right)^{3/2} \exp\left(\frac{EF - E_c}{KT}\right)$$

$$n_i = 2\left(\frac{2\pi m_e KT}{h^2}\right)^{3/2} \exp\left(\frac{E_i - E_c}{KT}\right)$$

Dividing n by n_i

$$n = n_i \exp\left(\frac{EF - E_i}{KT}\right) \text{-----(9)}$$

$$p = p_i \exp\left(\frac{E_i - EF}{KT}\right) \text{-----(10)}$$

$$p_i = n_i \Rightarrow p = n_i \exp\left(\frac{E_i - EF}{KT}\right) \text{-----(11)}$$

Multiplying the eqn (9) with (11) yields

$$np = n_i^2$$

- This expression is called as the law of mass action. It can be stated as “The product of free carrier concentrations at a given temperature is a constant for any type of semiconductor in equilibrium”.
- The product np is the probability of simultaneous existence of electron in the conduction band and holes in the valance band. N_i refers to thermally generated carriers concentration. Thus the rate of recombination of carriers and the rate of generation must be equal. The law of mass action is merely an outcome of this concept.
- As a consequence of equ (9) & (11) both n and p cannot exceed n_i simultaneously if $n > n_i \Rightarrow p < n_i \Rightarrow n > p$ and $n < n_i \Rightarrow p > n_i \Rightarrow p > n$ Thus n type semiconductors have $n > p$ and electrons are majority carriers. In p-type the holes form the majority carriers. If $n = p$ the concentration become equal to the intrinsic concentration.
- **Extrinsic Semiconductor minority carrier concentration:-** In highly doped n-type semiconductors as the acceptor atoms will not be there $N_A = 0$ and thus $N_{d^+} \gg n_i$ Hence $n = N_{d^+}$. If the donors are fully ionized then $N_d = N_{d^+} = n_n$ using law of mass action we can write p_n the hole equilibrium

concentration in n-type semiconductors which can be written as
$$P_n = \frac{n_i^2}{N_D}$$

in highly doped p-type semiconductors $N_{d^+} = 0$ the majority and minority

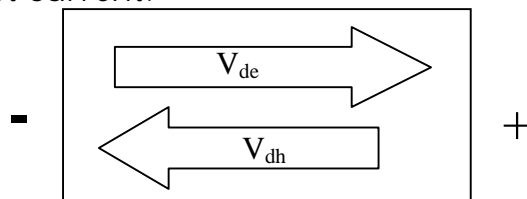
carriers in equilibrium are $p_p = N_{a^-} = N_A$ and
$$n_p = \frac{n_i^2}{N_A}$$

Minority carrier life time is defined as the time required to reduce initial concentration to $1/e$ its initial concentration.

S4. Drift and Diffusion currents –Einstein’s relations

Drift current and conductivity:

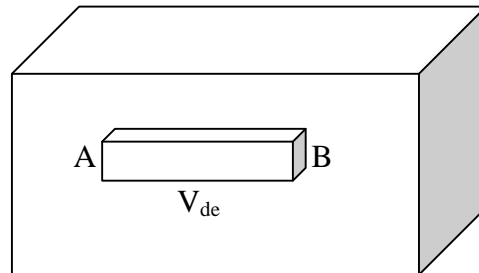
- The application of an electric field E causes a directed drift velocity (V_{de} or V_{dh}) to be superimposed on the free carriers. The flow of charged carriers will result in electric currents flowing in the sample. This mechanism of current flow is called drift current.



- Let us consider a homogeneous semiconductor sample having free electron concentration of n and a free hole concentration of P . An electric field E is applied in the positive x – direction causing drift currents. J_{ndrift} and J_{pdrift} denote respectively the current densities due to electron and hole drifts.

Before the field E is applied the n electrons move randomly with an rms velocity or V_{te} and a mean velocity of zero in any direction.

- The current density is therefore zero. When the field E is applied, the conduction electrons acquire a mean velocity of V_{de} . The current contribution by these n electrons will be the same as due to n electrons all moving with the same velocity V_{de} and a mean velocity of zero in any direction.



- Consider a small rectangular block within the crystal having a unit cross sectional area and length V_{de} in the x -direction. Let A and B be the crystal faces perpendicular to the flow of electrons such that AB represents the direction of motion of electrons. Then volume of the block = $V_{de} \times 1 \times 1 = V_{de}$
 Number of electrons in the block = nV_{de} charge contained in this volume = $-nqV_{de}$
- All of this charge contained in the block will in unit time cross the face B of the block furthermore no other free electron in the semiconductor outside this block will cross face B . Since current density is defined as the amount of charge crossing unit area per unit time the drift current density due to electrons at face B will be.

$$J_n \text{ drift} = -nqV_{de} \text{-----(1)}$$

By a similar line of argument we can conclude that $J_p \text{ drift} = pqV_{dh}$ also we know that $v_{de} = -\mu_e E$ Hence $J_n \text{ drift} = nq\mu_e E$, $J_p \text{ drift} = pq\mu_p E$

The total drift current due to both hole and electron flow is therefore

$$J_{\text{drift}} = J_n \text{ drift} + J_p \text{ drift} \\ = nq\mu_e E + pq\mu_p E \text{-----(2)}$$

Note that although the electrons and holes move in opposite directions in response to an impressed electric field E the effective direction of charge flow is the same and so the current components add.

- The ratio of the drift current to the apply field is called the conductivity and the equations.

$$J_{\text{drift}} = \sigma E \text{ comparing with equ(2)}$$

$$\sigma = nq\mu_n + pq\mu_p \Rightarrow \sigma_n + \sigma_p$$

In case of n-type semiconductor $n > p$

$$\sigma_n \gg \sigma_p \Rightarrow \sigma = \sigma_n$$

$$\text{p-type } p > n \quad \sigma_p \gg \sigma_n \Rightarrow \sigma = \sigma_p$$

Diffusion currents:

- Diffusion takes place due to the existence of a non-uniform concentration of carriers.
- In fact Fick's first law states that the diffusion flux F i.e., the particle current is proportional to and in a direction opposite to the concentration gradient of the particles. In one dimension this can be stated mathematically as

$$F = -D \frac{\partial N}{\partial x} \text{-----(1)}$$

Hence if n and p are the electron and hole concentrations then the particle flux densities of electrons and holes J_e and J_p are $J_e = -D_n \frac{\partial n}{\partial x}$ and

$J_p = -D_p \frac{\partial p}{\partial x}$. D_n & D_p are called the electron and hole diffusion constants respectively. Since each electron and hole carries a charge $-q$ and $+q$ respectively the electric current densities are

$$J_e = -q D_n \frac{\partial n}{\partial x} \text{ and } J_{n,diff} = q D_n \frac{\partial n}{\partial x}$$

$$J_p = -q D_p \frac{\partial p}{\partial x} \text{ and } J_{p,diff} = -q D_p \frac{\partial p}{\partial x}$$

When both drift and diffusion currents are present the total hole and electron current densities can be written as

$$J_n = J_{n,drift} + J_{n,diff} \text{ Or } J_n = nq\mu_n E + qD_n \frac{\partial n}{\partial x}$$

$$J_p = p q \mu_p E - q D_p \frac{\partial p}{\partial x}$$

Einstein Relation:

Although the carrier transports mechanisms drift and diffusion have been considered independently they are both dependent on the scattering process responsible for hindering the flow of carriers in semiconductors.

- Einstein showed that the parameters describing the two processes mobility μ and diffusion coefficient D respectively are directly related. At equilibrium with no applied electric field the free electron distribution is uniform and there is no current flow.
- Any tendency to disturb the state of equilibrium which would lead to a diffusion current creates an internal electric field and a drift current balancing the diffusion component under equilibrium conditions therefore the drift and diffusion currents due to an excess density of electrons (Δn)

$(\Delta n)qE\mu_n = qD_n \frac{\partial(\Delta n)}{\partial x}$. The force F on excess carriers restoring equilibrium is given by the product of excess charge and electric field.

$$F = (\Delta n)qE = qE = \frac{qDn}{\mu n} \frac{\partial(\Delta n)}{\partial x}$$

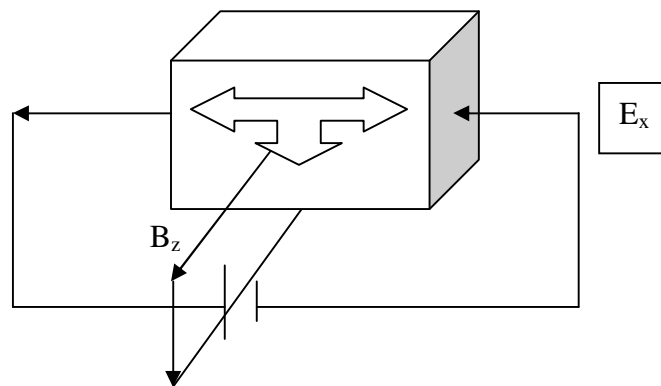
By making an analogy between excess carriers in a semi conductor and gas molecules in a low pressure gas (KT) $\frac{\partial(\Delta n)}{\partial x}$

$$KT = qD_n / \mu_n \Rightarrow D_n = \frac{KT}{q} \mu_n, D_p = \frac{KT}{q} \mu_p$$

The relation between diffusion coefficient and mobility of a charge carrier is termed Einstein's relation.

S4. Hall Effect

- The Hall effect arises out of an interaction between electric and magnetic fields in a semiconductor.
- Consider a rectangular semiconductor with carrier concentrations n and p placed at the origin of a Cartesian coordinate system x, y and z denote unit vectors in the three directions. An electric field E_x is applied in the x direction, the y direction of the sample is an open circuit and a magnetic field B_z is applied in the z direction.



- As a result of the crossed electric and magnetic fields the mobile particles both experience a Lorentz force F_l given by $F_l = e(vxB)$

Where e is the charge and v the velocity of the moving particle. The velocity in this case will be the drift velocity caused by the applied field E_x . As the drift velocities V_{de} & V_{dh} for electrons and holes in the x direction will be

$$J_{nx} = -nqv_{de} = nq(-V_{de})x = nq\mu_n E_x$$

$$J_{px} = qpV_{dh} = pq(V_{dh})x = pq\mu_p E_x$$

- J_{nx} and J_{px} are the electron and hole drift current densities. Notice that the opposite signs of charges of holes and electrons imply oppositely directed drift velocities V_{de} and V_{dh} . So the Lorentz forces F_e and F_h experienced by an electron and hole respectively is

$$F_e = -q(+V_{de} \times b_{zz})$$

$$F_e = qV_{de}B_z y$$

$$F_h = -qV_{dh}B_z y$$

- It is thus apparent that the Lorentz forces F_e and F_h acting on electron and holes both act in the same direction and tend to push the mobile carriers downwards.
- The field components E_{ye} and E_{yh} for electron and holes set up by the Lorentz forces will be

$$F_h = qE_{yh}$$

$$F_e = -qE_{ye}$$

$$E_{ye} = V_{de}B_z y \quad \& \quad E_{yh} = -V_{dh}B_z y$$

Since the oppositely charged electrons and holes are both deflected downwards the current densities J_{ye} and J_{yh} will be in opposite directions.

$$J_{ye} = \sigma_e E_{ye}$$

$$= nq\mu_n V_{de} B_z y$$

$$= nq\mu_n^2 E_x B_z y$$

$$\text{And } J_{yh} = \sigma_h E_{yh}$$

$$= pq\mu_p (-V_{dh}) B_z y$$

$$= -pq\mu_p^2 E_x B_z y$$

$$\text{Where } V_{de} = \mu_n E_x \quad \text{and} \quad V_{dh} = \mu_p E_x$$

- In steady state, under open-circuit conditions there can be no current in that direction. An opposing internal field E_H therefore builds up in the material to balance the net y direction current due to the Lorentz forces. This field is called the Hall field.

$$\text{Hence } \sigma E_H + J_{ye} + J_{yh} = 0$$

$$(pq\mu_p + nq\mu_n)E_H = -(nq\mu_n^2 - pq\mu_p^2)E_x B_z$$

$$E_H = \frac{(pq\mu_p^2 - nq\mu_n^2)E_x B_z}{pq\mu_p + nq\mu_n}$$

$$= \frac{(pq\mu_p^2 - nq\mu_n^2)J_x B_z}{(pq\mu_p + nq\mu_n)^2}$$

$$J_x = \sigma E_x$$

The ratio of the Hall field to the applied electric field is called Hall angle θ

$$\theta = \frac{E_H}{E_x} = \frac{(pq\mu_p^2 - nq\mu_n^2)B_z}{pq\mu_p + nq\mu_n}$$

The hall coefficient R_M is defined by the relation

$$R_M = \frac{E_H}{J_x B_z}$$

$$\text{Hence } R_M = \frac{pq\mu_p^2 - nq\mu_n^2}{(pq\mu_p + nq\mu_n)^2}$$

Note that in extrinsic p type materials ($p \gg n$)

$$R_M = \frac{1}{pq}$$

And in n type material ($n \gg p$)

$$R_M = -\frac{1}{nq}$$

The sign of the Hall coefficient thus depends on the type of doping in the semiconductor.

- **Direct and indirect band gap semiconductors**

Direct

- If the electron jumps directly from conduction band to valence band it is known as Direct band gap semiconductor.
- The transition occurs along $k=0$
- Optical transmission is possible

Ex. GaAs

Indirect

- If the electron jumps from conduction band to valence band via trap levels it is known as indirect band gap
- The transition occurs along $k \neq 0$
- Optical transmission not possible

Ex. GaP

Solid State Physics

UNIT - III

Assignment-Cum-Tutorial Questions on optical fiber

Section A: Objective Questions

1. The expression for the mobility of the electrons in terms of the Hall coefficient in n-type semiconductor _____.
2. Einstein's relation between drift and diffusion mechanisms is - _____.
3. In a material the diffusion current density is _____ to the concentration gradient of the charge carriers.
4. In an intrinsic semiconductor, the intrinsic conductivity depends on _____.
5. The mass action law can be written as _____.
6. In a semiconductor, the direction of motion of hole is _____ to that of the direction of motion of electron.
7. When the temperature of semiconductor is nearly 0 K, it behaves as good as _____.
8. At absolute temperature the Fermi level in an intrinsic semiconductor is _____.
9. Example for the direct band gap semiconductor is _____.
10. When trivalent impurities are doped into the pure semiconductor the concentration of _____ will be increased.
11. At T=0K the electrons above Fermi level is _____ below Fermi level is _____.
12. Fermi function is valid for _____ particles.
13. If $m_h^* > m_e^*$ then the position of Fermi energy level is : []
 - a) At the centre of energy gap
 - b) Just above the centre of energy gap
 - c) Just below the conduction band
 - d) Just above the valence band
14. When the temperature of semiconductor is nearly 0 K, it behaves as good as ----. []
 - a) Conductor
 - b) semimetal
 - c) insulator
 - d) None of the above
15. When an electric field is applied to the semiconductor, the current density in terms of mobility of the electrons. []
 - a) $J_n = ne\mu_e E$
 - b) $J_n = pe\mu_e E$
 - c) $J_p = ne\mu_h E$
 - d) None of the above
16. Which type of semiconductor satisfies the relation $N_A = p$? []
 - a) p-type semiconductor
 - b) n-type semiconductor
 - c) intrinsic
 - d) None of the above
17. Hall coefficient of a semiconductor is -----. []
 - a) $R_H = -\frac{1}{ne}$
 - b) $R_H = \frac{1}{e}$
 - c) $R_H = \frac{1}{n}$
 - d) None of the above

18. The emission wavelength of emitted light is given by ----. []
 a) $\lambda_g = \frac{hc}{E_g}$ b) $\lambda_g = \frac{p}{E_g}$ c) both a & b d) None of the above
19. The diffusion electron current density in an n-type of semiconductor is - []
 a) $J_n = eD_n \frac{dn}{dx}$ b) $J = -eD \frac{dx}{dn}$ c) $J = nqDn$ d) None of the above
20. When the diffusion mechanism occurs in a semiconductor? []
 a) Concentration gradient of charge carriers exists in a semiconductor
 b) Concentration is same throughout the semiconductor
 c) External field required
 d) None of the above
21. The mobility of electrons and holes in an intrinsic semiconductor are 0.8 and $0.18 \text{ m}^2/\text{V} - \text{s}$. Find the intrinsic conductivity if $n_i = 2.5 \times 10^{19}/\text{m}^3$. []
 a) $2.24(\Omega - \text{m})^{-1}$ b) $1.24(\Omega - \text{m})^{-1}$ c) $0.24(\Omega - \text{m})^{-1}$ d) None of the above
22. Find the electron mobility in n-type semiconductor if $R_H = 160 \text{ cm}^3/\text{Coulomb}$ and $\rho = 0.16 \text{ ohm} - \text{cm}$. []
 a) $2000 \text{ cm}^3/\text{ Volt-s}$ b) $1000 \text{ cm}^3/\text{ Volt-s}$
 c) $3000 \text{ cm}^3/\text{ Volt-s}$ d) None of the above
23. Find the ground state energy of an electron when it is confined to move in a one dimensional box of length 0.01 nm ($m = 9.1 \times 10^{-31} \text{ Kg}$ and $h = 6.625 \times 10^{-34} \text{ J} - \text{S}$). []
 a) $6.04 \times 10^{-18} \text{ J}$ b) $5.04 \times 10^{-18} \text{ J}$ c) $4.04 \times 10^{-18} \text{ J}$ d) None of the above

Section B: (Subjective Questions)

- a) What are drift and diffusion currents? Obtain Einstein relation between their coefficients 8M(CO3)

b) Show that $E_i = E_c + E_v / 2$ 4M(CO3)
- a) Derive an expression for the number of electrons per unit volume in the conduction band of N- type semiconductor. 6M(CO3)

b) Distinguish between direct and indirect band gap semiconductors 6M(CO3)
- a) Obtain an expression for the conductivity of an intrinsic semiconductor in terms of carrier concentration. 8M(CO3)

b) A silicon plate of thickness 1mm, breadth 10mm and length 100mm is placed in a magnetic field of 0.5 wb/m² acting perpendicular to its thickness. If 10⁻² A current flows along its length, calculate the Hall

- voltage developed if the Hall coefficient is $3.66 \times 10^{-4} \text{ m}^3/\text{coulomb}$.
- 4M(CO3)
4. Derive the expression for carrier concentration of holes and electrons in Intrinsic semiconductors. 10M(CO3)
5. a) Derive the carrier concentration in Intrinsic semiconductors. 10M(CO3)
- b) What are indirect band gap semiconductors? 2M(CO3)
6. a) Derive the carrier concentration in Intrinsic semiconductors. 10M(CO3)
- b) What are direct band gap semiconductors? 2M(CO3)

Section C: (Additional Questions)

- The following data are given for intrinsic germanium at 300K $n_i=2.4 \times 10^{19} /\text{m}^3$, $\mu_e=0.39\text{m}^2\text{V}^{-1}\text{s}^{-1}$, $\mu_p=0.19\text{m}^2\text{V}^{-1}\text{s}^{-1}$ calculate the resistivity of the sample.
- The resistivity of an an intrinsic semiconductor is 4.5 ohm-m at 20° C and 2.0 ohm-m at 32° C what is the energy gap?
- Find the diffusion coefficient of electron in silicon at 300K if $\mu_e=0.19\text{m}^2\text{V}^{-1}\text{s}^{-1}$
- The Hall coefficient of certain silicon specimen was found to be $7.35 \times 10^{-5}\text{m}^3\text{C}^{-1}$ from 100 to 400K. Determine the nature of the semiconductor, If the conductivity was found to be $200 \text{ m}^{-1}\Omega^{-1}$, calculate the density and mobility of the charge carrier.
- The R_H of a specimen is $3.66 \times 10^{-4}\text{m}^3\text{c}^{-1}$. Its resistivity is $8.93 \times 10^{-3} \Omega\text{m}$. Find the μ and n .
- In the following cases, show the position of the Fermi level at $T=0\text{K}$ in energy band diagram and give reason for your answer. Mark E_C , E_V , E_F , E_I , E_A , E_D for : a) N type, $N_A= 0$, b) P type, $N_D=0$.

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Solid State Physics

Unit-IV Dielectrics

Course objectives

- To analyze the interactions between atoms, molecules in solids to exhibit dielectric properties.

Syllabus:

Expression for local field - Clausius mosotti relation - Types of polarization - Frequency response curve of dielectrics - Dielectric loss - Dielectric strength - Loss tangent

Course Outcomes

Student will be able to

- apply the basic principles and understand the factors influencing the dielectric properties.
- understand the charge storage capacity of dielectrics.

DIELECTRICS

1. Introduction: Dielectrics are insulating materials. There are no free charge carriers in a dielectric. In dielectrics the forbidden energy gap is $> 3\text{eV}$. Such a large energy gap precludes thermal excitation of electrons from the valence band to the conduction band. Even externally applied fields fail to excite the electrons. Dielectrics find extensive use in electrical and electronic industries. They are used for insulation purposes. All capacitors use dielectric materials between their plates.

2. Terminology:

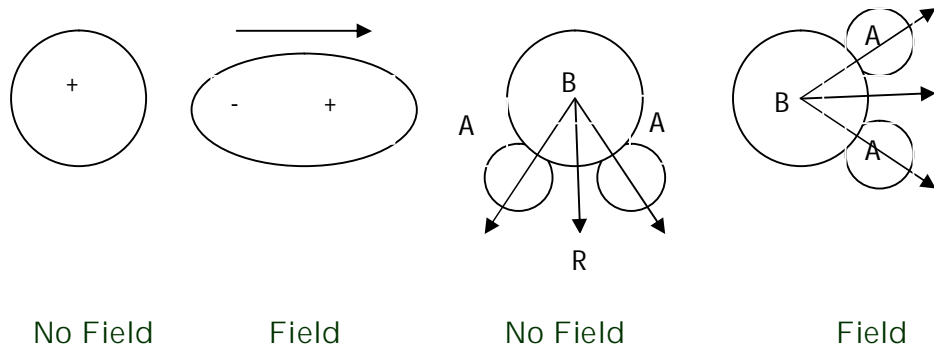
a. Dipole: Two equal and opposite charges (+q, -q) separated by a distance $2r$ constitute a dipole. The dipole moment is defined by $\mu = qr$. If each particle (atom/molecule) of a substance possesses dipole moment the dipole moment of the sample is

$$p = \sum_{i=1}^i q_i r_i$$

b. Non-Polar Dielectric: Monoatomic materials are made up of atoms. In general the centre of the negative charge and the centre of positive charge of an atom coincide. Even though there are two equal and opposite charges they are not separated. Their dipole moment $\mu = q \times 0 = 0$. Such dielectrics are called non-polar dielectrics.

c. Polar Dielectrics: Polyatomic substances contain molecules. The centre of the negative charge distribution may or may not coincide with the centre of the positive charge distribution. If there is an effective separation between the centers of the negative and positive charge distributions, the molecule has a dipole moment P . Such dielectrics are called polar dielectrics
Ex: Water Nitrobenzene.

d. Polarization: It is the process of inducing dipole moment in medium by an applied electric field. Polarization can take place both in polar and non-polar dielectrics.



In polar dielectrics molecules are oriented at random. As a consequence in any volume containing large no.of molecules the net dipole moment is zero. When a polar molecule is exposed to an external electric field each of the molecular dipoles experiences a torque. As a result they tend to orient themselves in the direction of field.

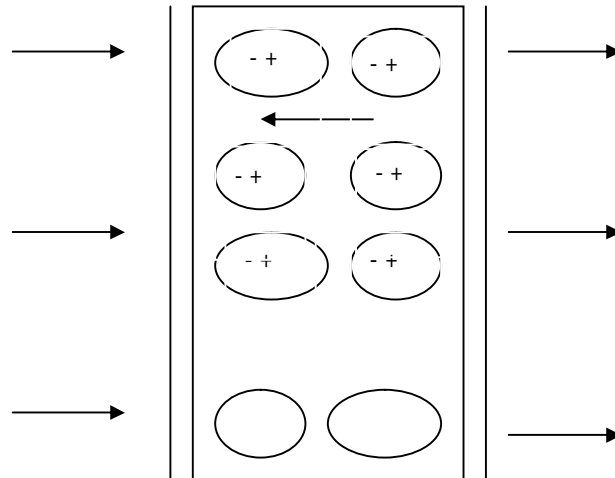
- e. **Surface Charge Density** (σ_p): Consider a rectangular slab of length l and area of cross-section A placed in an electric field. Let σ_p be the induced charge density. Then polarization

$$p = \frac{\text{dipolemoment}}{\text{volume}} = \frac{\sigma_p Al}{Al} = \sigma_p \text{ i.e}$$

The induced surface charge density is equal in magnitude to the polarization p .

- f. **Dielectric Constant (or relative permittivity)**: The dielectric constant also called relative permittivity is defined as

$$k = \epsilon_r = \frac{\text{Electric field in a region without the dielectric}}{\text{Electric field with the dielectirc.}}$$



Consider a dielectric slab placed in an electric field E_0 the dielectric gets polarized. As a result an extra electric field E_p is developed. The resultant field $E = E_0 - E_p$

The dielectric constant $K = \frac{E_0}{E_0 - E_p} = \frac{E_0}{E}$ or the electric field within

the dielectric $E = E_0 / K$ Hence dielectric constant of a material is the factor by which the electric field is reduced.

g. **Polarizability**:- The dipole moment induced in an atom is found to be proportional to the local electric field E_L $\mu \propto E_i \Rightarrow \mu = \alpha E_i$
 The constant of proportionality α is called the polarizability. If there are N molecules per unit volume the polarization of the solid

$$P = N\mu = N \alpha E_i$$

h. **Susceptibility**: Polarization per unit electric field is called electric susceptibility $\chi_e = P / \epsilon_e E$

3. 1 Local or Internal Field :- "The electric field which a dipole experiences in a medium is called the Local field or the internal field (E_i) it is different from the externally applied field E₀"
 Consider a dielectric subjected to an external field E₀ Imagine a small spherical cavity of radius r with its centre at the dipole chosen for consideration. The dipole experiences the following fields, in addition to the externally applied field E₀.

- a. The field E₁ produced by charges on the specimen this is proportional to polarization P.
- b. The field E₂ arising from dipoles inside the sphere. The value of E₂ depends on the crystals E₂=0
- c. The field E₃. It is due to polarization of charges on the surface of a fictitious cavity cut out of the specimen with reference atom as center it is called the Lorentz cavity field. Let us now calculate E₃. The cavity is cut out of the medium of polarization P. If θ is the polar angle referred to the polarization direction, the surface charge density on the surface of the cavity is $P \cos \theta$. If ds is the area of the surface element the charge on the surface element is = (normal component of polarization) (area of the surface element) = $P \cos \theta ds$

A test charge 'q' placed at the centre of the cavity experiences a force according to coulomb's law

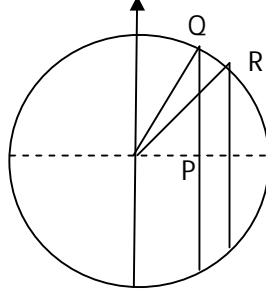
$$dF = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2} = - \frac{1}{4\pi \epsilon_0} \frac{q P \cos \theta ds}{r^2}$$

According to coulomb's law $dE_3 = \frac{dF}{q} = - \frac{1}{4\pi \epsilon_0} \frac{P \cos \theta ds}{r^2}$

The electric field can be resolved into two components: One component along the direction of P and the other perpendicular to it. The perpendicular components cancel themselves out leaving only the horizontal ones.

The sum of all such components for the whole surface is given

$$\text{by } \int dE_3 = \frac{1}{4\pi \epsilon_0} \int \frac{(P \cos \theta)(\cos \theta) ds}{r^2}$$



The surface area of the ring considered is given by $ds = (2\pi r \sin \theta) r d\theta = 2\pi r^2 \sin \theta d\theta$.

Substituting for ds and integrating between the limits 0 and π we get

$$E_3 = \frac{1}{4\pi \epsilon_0} \int_0^\pi \frac{P \cos^2 \theta}{r^2} 2\pi r^2 \sin \theta d\theta \rightarrow E_3 = \frac{P}{2 \epsilon_0} \int_0^\pi \cos^2 \theta \sin \theta d\theta$$

Let us now evaluate the above integral by putting

$$\cos \theta = z \text{ then } dz = -\sin \theta d\theta$$

$$\text{then } E_3 = \frac{P}{2 \epsilon_0} \int_1^{-1} z^2 dz$$

$$\therefore E_3 = \frac{P}{3 \epsilon_0}$$

Total internal field may be expressed as $E_1 = E_0 + E_1 + E_2 + E_3$

$$= E_0 + E_1 + E_3 \quad (E_2=0)$$

$$E_1 = E + \frac{P}{3\epsilon_0}$$

Since E_0 & E_1 act along the same line $E_0 + E_1$ can be expressed by E .

3.2 Clausius – Mosotti equation:-

Let us consider now the simple case of an elemental solid dielectric which exhibits only electronic polarizability. If α_e is the electronic polarizability per atom it is related to bulk polarization P through the relation $P = N \alpha_e E_i$

$$\alpha_e = \frac{P}{NE_i} \Rightarrow \frac{P}{N[E + P/3\epsilon_0]} \quad \because E_i = E + P/3E_0$$

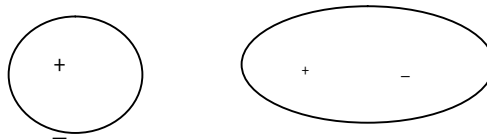
But $P = x \epsilon_0 E$ $X = \epsilon_r - 1 \Rightarrow E = \frac{P}{\epsilon_0(\epsilon_r - 1)}$ using E value

$$\alpha_e = \frac{P}{N \left[\frac{P}{\epsilon_0(\epsilon_r - 1)} + \frac{P}{3\epsilon_0} \right]} \Rightarrow \frac{N\alpha_e}{\epsilon_0} = \frac{1}{\left[\frac{1}{\epsilon_r - 1} + \frac{1}{3} \right]} = \frac{1}{\left[\frac{\epsilon_r + 2}{3(\epsilon_r - 1)} \right]}$$

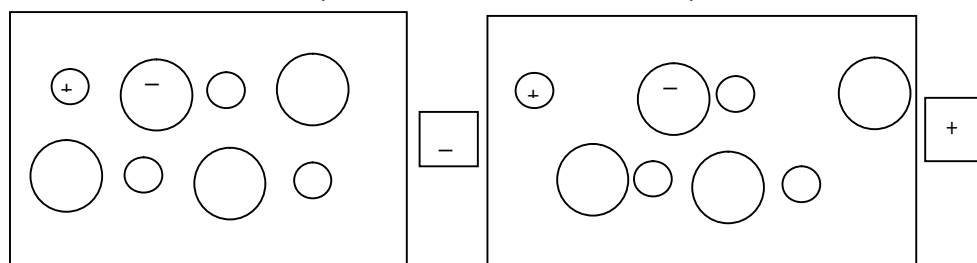
$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_e}{3\epsilon_0} \quad \alpha_m = \frac{M}{\rho} \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\rho\alpha}{3\epsilon_0}$$

4. **Types of Polarization:-** Polarization is the process of inducing dipole moment in a molecule.

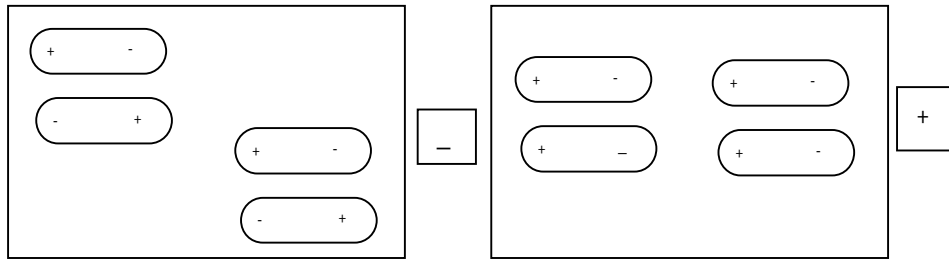
i) **Electronic polarization (α_e)** it is due to displacement of the negative electron cloud of each atom with respect to its nucleus. It occurs in all dielectrics without an external electric field where the centre of the positive charge and the centre of the negative distribution coincide. As such there is no dipole moment. When an external field is applied separation of the centers of negative and positive charges leads to development of dipole moment.



ii) **Ionic Polarization (α_i):-** This occurs in ionic dielectrics like NaCl when such a material is subjected to opposite sign undergo displacement. This displacement causes an increase or decrease in the distance of separation between ions depending on their location. Such a separation leads to a net dipole moment.



iii) **Orientation or Dipolar Polarization (α_0/α_d)** this type of polarization occurs in materials with polar molecules without external field the molecules are oriented at random. The material exhibits net zero dipole moment.

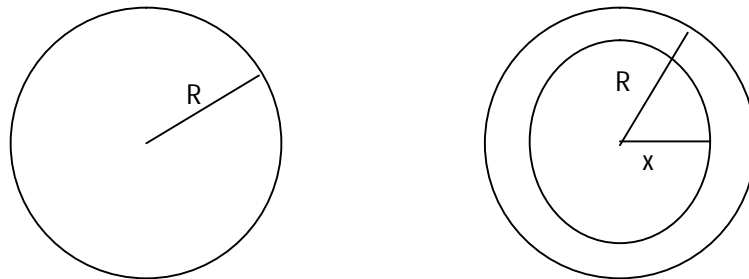


The total polarization α of a material is given by the sum of the electric, ionic

and dipolar polarizabilities.

$$\alpha = \alpha_e + \alpha_i + \alpha_0$$

4.1 Electronic Polarizability > Estimation of ' α_e ' Consider an atom in the absence of an external field. The centers of the positive and negative charges coincide leading to zero dipole moment. But when an external field E is applied the nucleus and the electron cloud will tend to move in opposite directions. However a force will develop between them which tends to drive the charges back to their normal positions. As a consequence an equilibrium is established in which the nucleus is displaced by an amount x relative to the electron cloud.



The force on the nucleus in the field direction is ZeE . The electron cloud can be imagined to be in two regions one inside a sphere of radius x and the other between two spherical surfaces of radii x and R . According to Gauss theorem, the charge in the second region does not exert a force on the nucleus. The negative charge inside the sphere of radius x only exerts a force of $\frac{-Zex^3}{R^3}$ on the nucleus.

Applying coulomb's Law, the force between the nucleus and the negative charge within a sphere of radius x is given by

$$F = \frac{1}{4\pi\epsilon_0} \frac{Ze(-Zex^3/R^3)}{x^2} \text{-----(A)}$$

$$\text{But } F = ZeE \Rightarrow F = ZeE = \frac{1}{4\pi\epsilon_0} \frac{Ze(-Zex^3)}{x^2} \Rightarrow x = -4\pi\epsilon_0 R^3 E / Ze$$

The resulting dipole moment $\mu = Zex = 4\pi\epsilon_0 R^3 E$

The electronic polarizability $\alpha_e = \frac{\mu e}{E} = -4\pi\epsilon_0 R^3$

Ionic Polarizability (α_i):- Expression for α_i :- The ionic polarization is due to displacement of cations and anions in opposite directions in an ionic solid. If x_1 and x_2 are displacements of the positive and negative ions respectively the induced dipole moment per unit cell is

$$\mu = e(x_1 + x_2)$$

Restoring forces come into play due to these displacements under equilibrium conditions the restoring force $F = K_1 x_1 = K_2 x_2$ (K_1, K_2 are force constants) $x_1 = F/K_1 = eE/m\omega_0^2$ ($K_1 = m\omega_0^2$) (E applied field, m mass of the anion) For the cation of mass M $x_2 = F = eE/M\omega_0^2$

$$\text{The total displacement } x = x_1 + x_2 = \frac{eE}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right)$$

$$\text{And } \mu = e(x_1 + x_2) = \frac{e^2 E}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right)$$

$$\text{The ionic polarizability } \alpha_i = e^2 / \omega_0^2 \left(\frac{1}{M} + \frac{1}{m} \right)$$

Here ω_0 is the natural frequency of the ionic molecule for NaCl, $\omega_0 \cong 10^{13}$ rad⁻¹s i.e in the infrared region.

Dipolar Polarizability (α_d)

Orientalional Polarization α_d :- Polar dielectrics exhibit orientational or dipolar polarizability. Even in the absence of the external electric field polar dielectrics exhibit dipole moment when exposed to an external field the field tries to align these dipoles along its direction. As a result orientational polarization takes place.

Expression for αd :- when a molecule of dipole moment μ is exposed to an electric field E . it acquires a potential energy

$$n(\theta)d\theta = A.2\pi \sin \theta d\theta \exp\left[\frac{\mu E \cos \theta}{KT}\right]$$

Where A is a constant

The polarization is $Pd = N\bar{\mu} = N \int \frac{\mu \cos \theta 2\pi \sin \theta d\theta \exp\left[\frac{\mu E \cos \theta}{KT}\right]}{\int_0^{2\pi} 2\pi \sin \theta d\theta \exp\left[\frac{\mu E \cos \theta}{KT}\right]}$

In the above equation N is the no. of dipoles per unit volume.

Let $\cos \theta = x$ and $\frac{\mu E}{KT} = a$

$$P = N\mu \frac{\int_{-1}^{+1} x e^{ax} dx}{\int_{-1}^{+1} e^{ax} dx} = N\mu \frac{d}{da} \ln \int_{-1}^{+1} dx = N\mu \left[\frac{d}{da} \ln(e^a - e^{-a}) - \frac{d}{da} \ln a \right] = N\mu \left[\coth a - \frac{1}{a} \right] = N\mu L(a)$$

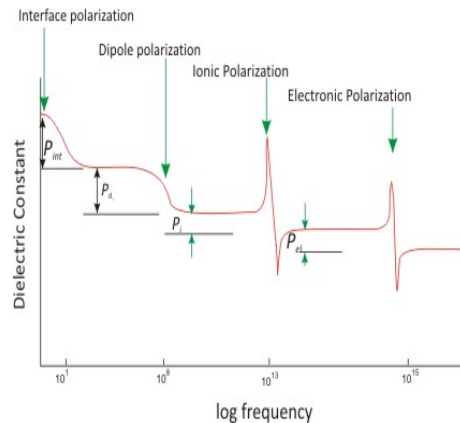
$L(a)$ is called the Langevin function

In practice $a \approx 10^{-4}$ and $L(a) \approx a/3$ so that

$$P_0 = N\mu \frac{\mu E}{3KT} = N\bar{\mu} \quad \text{where } \bar{\mu} \text{ is the average dipole moment in the}$$

directions of the field. The dipolar polarizability $\alpha d = \frac{\bar{\mu}}{E} = \frac{\mu^2}{3KT}$

Frequency Dependence of Total Polarizability:



The total polarizability of dielectric material is given by

$$\alpha_T = \alpha_d + \alpha_i + \alpha_e = \frac{\mu^2}{3KT} + \frac{e^2}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right) - 4\pi\epsilon_0 R^3$$

On application of an electric field any polarization process occurs as a function of time. The polarization at any instant of the t is given by $p(t) = P[1 - \exp(-t/\tau_r)]$ where P is the maximum polarization attained and τ_r is called the relaxation time for the particular polarization process. The relaxation time τ_r is a measure of the time scale of the process. It is the time taken for a polarization process to reach 63% of its maximum value.

The relaxation times vary widely for different polarization processes. Electronic polarization is extremely rapid it is essentially at the instant voltage is applied. Hence electronic polarization exists at all frequencies even when the frequency of the applied voltage is in the optical range ($\cong 10^{15}$ Hz)

Ionic polarization is slower than electronic polarization here the field has to displace much heavier ions, as compared to the electron cloud. The frequency with which can be displaced will $\cong 10^{13}$ Hz. If a field of frequency of $\cong 10^{15}$ Hz is applied to an ionic substance the ions do not respond as they need much longer time.

As a result, ionic polarization does not take place at optical frequencies. Ionic polarization takes place at frequencies lower than 10^{13} Hz.

Orientation of the dipoles in external fields takes comparatively much longer time i.e. in the range micro to nanoseconds. As a result orientation polarization takes place below the mega hertz range of frequencies.

DIELECTRIC BREAKDOWN

A dielectric material can retain its insulating property, only when the electric field to which it is subjected to, is of ordinary strength, at which time it can effectively block the flow of current between the two electrodes in which it is bound. But, when the field strength becomes very high of the order of few million volts/meter they can no longer withstand the strong pulling force exerted by the electric field on the polarization charges. The charges are ripped apart from the parent molecules and get swept by the field resulting in a great surge of current through the material. The resistance offered by the material to the flow of current crashes, and it becomes a good conductor of electricity. This state of material is called dielectric breakdown.

The dielectric breakdown can be explained on the basis of energy band diagram. For dielectric material, the conduction band is completely empty, and the valance band is completely filled, and two are separated by a large energy gap (of the order of 3eV or more). Flow of current is possible only when the electrons are lifted to the conduction band. Since the energy gap is very large, it needs a very high voltage to energize electrons

to jump the energy gap and enter into the conduction band. However, if this energy is achieved by applying a very high electric field, then all of a sudden a large number of electrons land up in conduction band, which also leave vacancies in the valence band, thus making conduction amenable in valence band too. Because of the sudden start of free flow of charges in the material, it is virtually turned into a good conductor and its ability to resist the flow of current breaks down.

Thus the dielectric breakdown is defined as follows

"The dielectric breakdown is the state of dielectric material subjected to a very high electric field, under the influence of which, the electrons are lifted into the conduction band causing surge of current, and the ability of the material to resist the current flow suffers a breakdown".

DIELECTRIC STRENGTH

Each dielectric material is identified with its ability to withstand very high electric fields. In this regard, dielectric strength is defined as follows

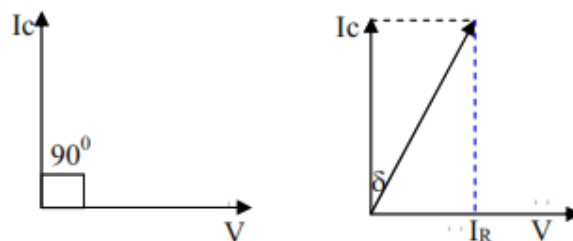
"Dielectric Strength is the maximum electric field that the dielectric material can withstand, without suffering electrical breakdown".

for example, we may note that the dielectric strength value for dry air is 3×10^6 V/m at 1 atmospheric pressure. 14×10^6 V/m for Pyrex glass, 60×10^6 V/m for Teflon.

DIELECTRIC LOSS

When a dielectric material is subjected to the influence of an electric potential, its constituent molecules come into a state of electrostatic stress due to polarization. The molecules behave as electric dipoles. If the polarity of the applied voltage is reversed, the stress is also reversed, because of which the dipoles switch their orientation. In order to do so, the dipoles are required to overcome a sort of internal friction which involves a loss of energy. This occurs every time the switching or the realignment of dipole takes place. The energy loss due to friction is always dissipated as heat by the dielectric to the surroundings. This is termed as dielectric loss, and defined as

" It is the loss of energy in the form of heat by a dielectric medium due to the internal friction developed as a consequence of switching action of molecular dipoles under certain a.c. conditions".



We Know that, when an alternating current flows through a resistor, the current I_R will be in phase with the applied voltage. But, when a capacitor is subjected to an alternating e.m.f., the current I_c flowing through the capacitor will lead the applied voltage by a phase angle of 90° . When an a.c. is applied across a capacitor with a dielectric, if I_R is the current through the equivalent resistance and I_c is that through capacitor then I_R is in phase with V and I_c leads V by 90° . The resultant of I_c and I_R is I . The angle between I and I_c is denoted by δ and is called the dielectric loss angle.

UNIT-IV

Assignment –cum- tutorial questions

Questions testing the remembering / understanding level of students

I. Objective

- The polarization that depends on the volume of the atom
 - Electronic polarization
 - Ionic polarization
 - Orientalional polarization
 - Space charge polarization
- _____ Molecules tend to orient themselves in an external electric field.
- Identify a dielectric material
 - Silicon
 - Tin
 - Rubber
 - Mica
- The temperature dependent polarization is
 - Electronic polarization
 - Ionic polarization
 - Orientalional polarization
 - All the above
- Dielectric Strength of a material is
 - the capacity to take two or more stress
 - the capacity with stand higher voltages
 - the capacity to withstand electrical mechanical shocks
 - none
- Choose the best insulator
 - SiO_2
 - PVC
 - Bakelite
 - Porcelain
- All insulators are dielectrics. (T/F)
- Any insulator stores electric charge. (T/F)
- In polar dielectrics, the dipoles align in single direction after switching off the external field. (T/F).
- How does the capacitance of a capacitor vary on introduction of dielectric?
 - Increases
 - No changes
 - Decreases
 - None
- The expression for Clausius – Mossotti equation is _____

II. DESCRIPTIVE QUESTIONS

- Show that in a dielectric, the local field experienced by an atom is greater than the applied field.
- What are the differences between insulators and dielectrics?
- Define Dielectric strength, Dielectric loss.
- Derive the relation between dielectric constant and susceptibility.
- Derive Clausius - Mossotti equation.
- What is the temperature dependent polarization? Write the relation.
- What are the differences between polar and non polar dielectrics?
- Derive an expression for electronic polarizability in dielectrics.
- Show that the ionic polarizability varies inversely with the frequency of vibrating molecule.
- Draw the frequency response curve for polizations in dielectrics

Problems:

1. The parallel plates of a capacitor have an area of $1 \times 10^{-1} \text{m}^2$ each and are $1 \times 10^{-2} \text{m}$ apart. A thin insulating plastic sheet is inserted between the capacitor plates. The potential difference drops from 2.00 kV to 1.00 kV between the plates whereas the charge on each plate remains constant. Calculate
 - a. Original capacitance
 - b. Magnitude of charge "q" on each plate
 - c. The capacitance 'C' after the dielectric is introduced.
 - d. Dielectric constant and permittivity.
2. If an electric field of magnitude 10^2N/C introduces a displacement of $1.0 \times 10^{-6} \text{m}$ between the electron cloud and the nucleus, what is the electronic polarisability thus in the material having an atomic number Z?
3. A monatomic gas contains 3×10^{25} atoms/ m^3 at 1 atmospheric pressure and at room temperature. The radius of gaseous atoms is 0.2 nm. Find the dipole moment per unit electric field, and dielectric constant.
4. The polarisability of Ne gas is $0.35 \times 10^{-40} \text{F m}^2$. If the gas contains 2.7×10^{25} atoms m^{-3} at 0°C and 1 atmospheric pressure, calculate its relative dielectric constant.

Solid State Physics

UNIT – V

Free electron and band theory of metals

Objective:

- To understand the behavior of electrons in the presence of external thermal and electrical energies.
- To gain insight into the effect of periodic potential of the lattice on electrons.

Syllabus:

Free electron and band theory of metals Classical free electron theory - Drawbacks of classical free electron theory – Fermi level and Fermi Dirac energy distribution function - Quantum free electron theory – Band theory-Bloch function - Kronig -penney model

Learning Outcomes:

At the end of this unit students are able to

- differentiate between classical and quantum free electron theories
- understand the interaction of electrons with ions in solids

Classical Free electron theory of Metals:

Metals achieve structural stability by letting their valance electrons roam freely through the crystal lattice. These valance electrons are the equivalents of the molecules of an ordinary gas. Since electrons are negatively charged particles their motion corresponds to a flow of electricity or electric current. It is assumed that the electrons move about at random and collide frequently with the residual ions. Hence the laws of classical kinetic theory of gases can be applied to a free electron gas also. Thus the electrons can be assigned mean free path λ , mean collision time τ and an average speed \bar{c} . In the absence of an externally applied potential difference, there are on an average as many electrons wandering through a given cross-section of the conductor in one direction as there are in the opposite direction. Hence the net current is zero. In between two collisions, the electron may move with the uniform velocity. During every collision both the direction and magnitude of the velocity get changed in general.

In 1900, P.Drude made use of the electron gas model to explain theoretically electrical conduction in metals. According to his theory the kinetic velocities of the electrons are assumed to have a root mean square velocity \bar{c} given by kinetic theory of gases in conjunction with the law of equipartition of energy \bar{c} is obtained as follows for unit volume of the metal.

$$P = \frac{1}{3} \rho \bar{c}^2 = \frac{1}{3} \frac{m N_A \bar{c}^2}{V_m}$$

Where V_m is molar volume and N_A is Avogadro's number.

$$\text{Now } PV_m = \frac{1}{3} m N_A \bar{c}^2 = R_U T \quad \text{Now Thus } m \bar{c}^2 = \left[\frac{3 R_U T}{N_A} \right] = 3 K_B T$$

Where R_U and K_B are universal gas constant and Boltzmann's constant respectively. The kinetic energy of the electron is

$$\frac{1}{2} m \bar{c}^2 = \frac{3}{2} K_B T$$

And $\bar{c} = \sqrt{\frac{3 K_B T}{m}} \quad \text{----- 1}$

$$\text{At } 20^\circ\text{C} \quad \bar{c} = \left[\frac{3 \times 138 \times 10^{-23} \times 293}{9.11 \times 10^{-31}} \right]^{\frac{1}{2}}$$

$$\bar{c} = 1.154 \times 10^5 \text{ m/s.}$$

Equation (1) indicates that the root mean square velocity of the electron is directly proportional to the square root of the absolute temperature of the metal.

At room temperature the drift velocity imparted to the electrons by an applied electric field is very much smaller than the average thermal velocity. The time τ taken by the electrons in traversing the distance λ

Will thus be decided not by the drift velocity due to the field but by the much greater velocity \bar{c} due to the kinetic thermal motion now

$$\tau = \frac{\lambda}{\bar{c}} = \lambda \left[\sqrt{\frac{m}{3K_B T}} \right]$$

the vital test of the validity of any theory of electrical conductivity is whether or not it can account for ohm's law. This law is usually expressed as $V = RI$. Where I is the current flowing through a conductor as the result of applying a potential difference V across its ends, at a given temperature R the resistance is a constant for a given conductor. In order to simplify the analysis let us rewrite

the law in terms of the current density $J = \frac{I}{A}$ and field strength along the conductor $E = -dv/dx$.

Thus we have

$$\frac{V}{R} = I = AJ$$

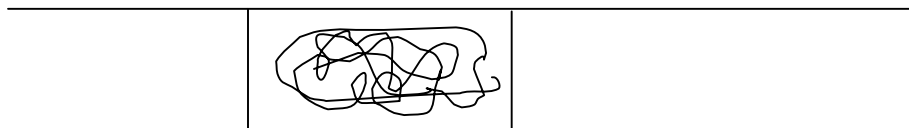
$$\frac{I}{J} = \frac{AR}{V} = \frac{Ael}{AEI} = \frac{\rho}{E}$$

$$J = \frac{E}{\rho} = \sigma E$$

Where the resistivity ρ and conductivity σ of a conductor of length l and area of cross-section A are defined by

$$P = \frac{1}{\sigma} = \frac{RA}{l}$$

Let us now assume that there are 'n' free electrons per cubic meter and that in the absence of any applied field these are darting about in all direction with no net velocity just like gas molecules in a container. When a field E_x is applied in the



X-direction all electrons are accelerated in the x-direction, with the acceleration of the i^{th} electron a_{ix} given by

$$a_{ix} = -\left[\frac{eE_x}{m}\right]$$

Alternatively this equation may be written as

$$\left[\frac{dv_{ix}}{dt}\right]_{E_x} = -\left[\frac{eE_x}{m}\right]$$

Where v_{ix} is the x-component of the velocity of the i^{th} electron, and the subscript E_x means the acceleration arising from the applied field. Since the right-hand side of this equation is the same for all electrons we may write above equation in another form

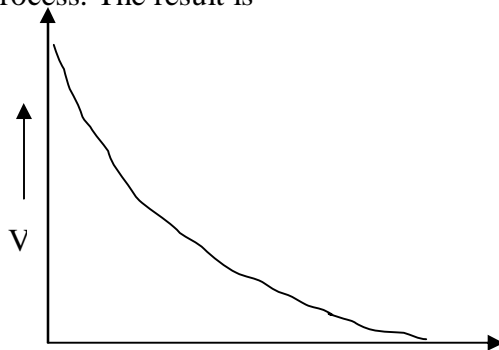
$$\left[\frac{d}{dt} \langle v_x \rangle\right]_{E_x} = -\left[\frac{eE_x}{m}\right] \quad \text{-----2}$$

Where $\langle v_x \rangle$ is the average velocity of all n electrons as given by

$$\langle v_x \rangle = \frac{1}{n} \left[\sum_{i=1}^n v_{ix} \right]$$

We have used $\langle v_x \rangle$ to denote the average value of v . The current density J is the charge flowing through unit area in unit time τ is a constant of the system known as the relaxation time.

It is in fact, the time taken for the drift velocity to decay to $1/e$ of its initial value Since $E_x = 0$ during this relaxation process we see that equation (5) can be used to obtain electron – lattice term in equation (4) and that this is done in such a way as to avoid, having to consider any details of the collision process. The result is



$$\left[\frac{d}{dt} \langle V_x \rangle\right]_{el-la} = -\frac{\langle V_x \rangle_0 e^{-t/\tau}}{\tau} = -\frac{t}{\tau} \quad \text{-----6}$$

If we assume that this expression is unchanged by the presence of a field.

We have from equations (5 & 6)

$$-\left(\frac{e}{m}\right)E_x - \frac{\langle V_x \rangle}{\tau} = 0 \quad \langle V_x \rangle = -\left[\frac{e\tau E_x}{m}\right]$$

This steady average velocity is imposed by the field and proportional to 'E' is called the drift velocity of the electrons. The constant of proportionality $\frac{e\tau}{m}$ is known as their mobility, μ and

the drift velocity from unit electric field is given by

$$\mu = \frac{\langle V_x \rangle}{E_x} = \frac{e\tau}{m}$$

The units of μ are meter per second per volt per meter or $m^2 v^{-1} S^{-1}$ Finally, we see that by combining equations

$$J = \frac{E}{P} = \sigma E, J_x = n \langle V_x \rangle (-e) = -nev_x$$

$$\text{and } \langle V_x \rangle = -\left[\frac{e\tau E_x}{m} \right]$$

$$\sigma = \frac{ne^2\tau}{m} \quad \text{or} \quad \rho = \frac{m}{ne^2\tau}$$

$$\sigma = ne\mu \quad \text{or} \quad \rho = \frac{1}{ne\mu}$$

This last equation is of great importance in the theory of conductivity and embodies the difference between the electrical properties of metals and semi-conductors. In metals where all the valence electrons are free at all temperatures n is a constant and the temperature variation of σ is essentially the temperature variation of μ . In semi-conductors, however n may also vary with temperature and it is usually the more important factor.

Fermi-Dirac Distribution (Non-Mathematical Treatment):

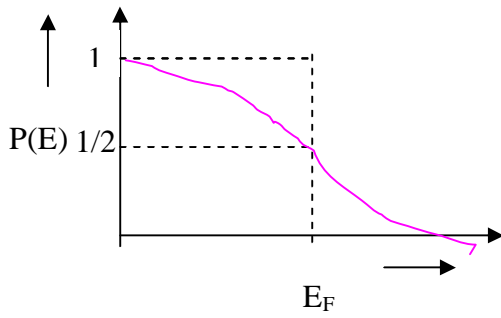
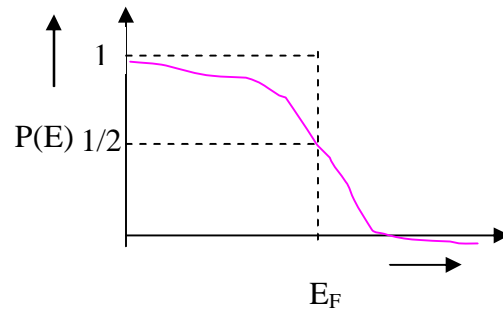
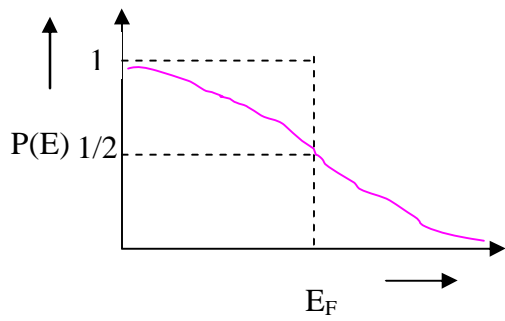
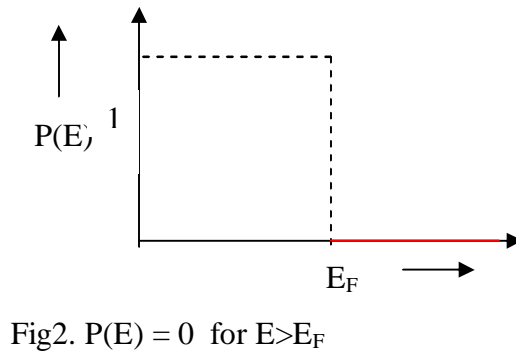
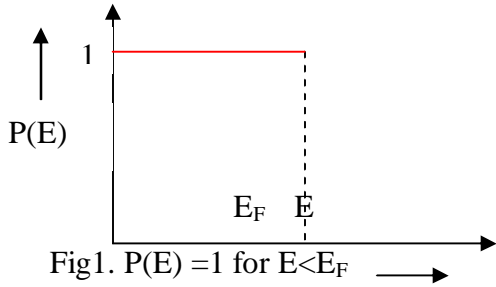
The Fermi Dirac probability distribution of energies has the form

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_f}{k_B T}\right)}$$

At T = 0K and for $E < E_F$, $f(E) = 1$

And for $E > E_F$, $f(E) = 0$.

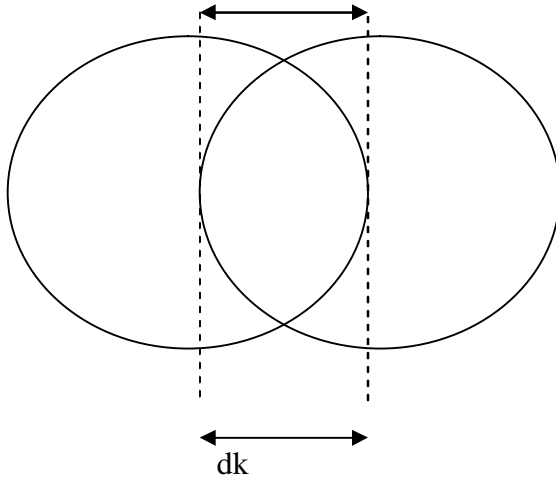
This means that at absolute zero, all the quantum states below E_F are occupied while all the quantum states having energies greater than E_F are unoccupied. So E_F is the maximum energy of the filled state. The Fermi function plot at a temperature greater than zero shows very little change from the Fermi plot at T= 0K only very near E_F , $f(E)$ begins to fall and at $E = E_F$, $f(E) = 1/2$ So Fermi level is that level at which the probability occupation is $1/2$.



Electrical conduction – Quantum Free electron Theory:-

According to the quantum free electron model, at 0K energy levels up to the Fermi level are filled. When the metal is not under the influence of an external field all the levels above the Fermi level are empty, those lying below the Fermi level E_F are completely filled.

On this basis, electrons at 0K fill a sphere of radius k_f in wave number space. k_f is the Fermi wave vector which corresponds to the maximum energy E_F . When there is no external electric field there is no net electron flow in any direction. Hence Fermi Surface is a sphere with its centre as origin



When an External electric field E is applied, each electron acquires additional velocity dv . This can be represented by the displacement of the Fermi sphere by dk . Only electrons near the Fermi surface contribute to conduction. Therefore

$$F = -eE = m \frac{dv}{dt} \quad [\because v = \frac{\hbar k}{m}]$$

$$\frac{dv}{dt} = \frac{\hbar}{m} \frac{dk}{dt} = \frac{m\hbar}{m} \frac{dk}{dt} = \hbar \left(\frac{dk}{dt} \right)$$

$$\text{or } dv = \frac{-eE}{m} dt$$

This additional velocity was acquired in a characteristic time τ

Therefore Current density $J = -nev_d$

$$= \frac{ne^2\tau}{m} E$$

The electrical conductivity is given by

$$\sigma = \frac{J}{E} = \frac{ne^2\tau}{m}$$

This expression is the same as that obtained on the basis of the Classical theory.

Even though both the classical and quantum theories lead to the same expression there is an essential difference in their approaches. According to the classical theory all free electrons contribute to electrical conduction. However according to the quantum theory only those electrons near the Fermi Surface take part in electrical conduction.

Band Theory of Solids

Let us examine the one-dimensional lattice. It consists of an array of ionic cores along x-axis. The potential of the electron at the core is zero while that in between the cores is maximum. The potential variation along any straight line passing through the centers of the ion cores is periodic. The periodicity of the potential is same as that of the lattice. The Schrodinger equation for the motion of the electron is given by

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}[E - V(x)]\psi = 0 \text{-----(1)}$$

The potential V(x) is of the form

V(x) = V(x+a) where 'a' is the periodicity of the lattice.

Bloch Theorem:

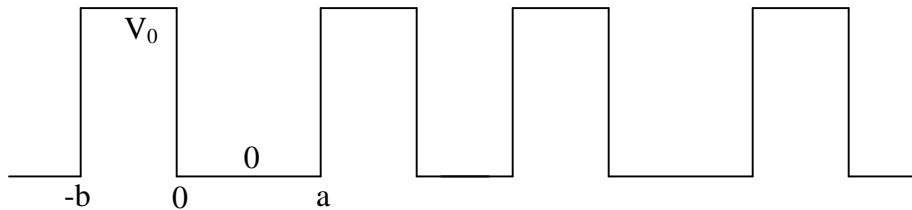
Bloch has shown that equation ---- (1) has a solution of the form

$$\psi_k(x) = U_k(x) e^{\pm ikx} \text{-----(2)}$$

In other words the solutions are plane waves $\psi(x) = e^{\pm ikx}$ modulated by a function $U_k(x)$ which depends in general on the wave vector 'k' and have the periodicity of the potential. This theorem is known as the Bloch theorem and the function (2) as the Bloch functions.

The Kronig – Penney Model:

The idealized periodic field assumed by Kronig and Penney is as shown in the figure



The potential wells of zero potential energy of width 'a' are separated by rectangular barriers of width 'b' and height V_0 . Each of the potential energy wells may be treated as a crude approximation of the potential in the vicinity of the ion cores. Although such model looks very simple, it explains the behavior of electrons in a crystal.

Solution of the wave equation: The Schrödinger wave equation for one dimensional periodic potential field denoted by V(x) is given by

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}[E - V(x)]\psi = 0 \text{-----(1)}$$

The above equation has to be solved for two regions.

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}E\psi = 0 \text{ for } 0 < x < a \text{-----(2)}$$

and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V_0)\psi \text{ for } -b < x < 0 \text{ ----- (3)}$$

$$\text{Let } \alpha^2 = \frac{8\pi^2 mE}{h^2} \text{ ----- (4)}$$

$$\text{and } \beta^2 = \frac{8\pi^2 mE}{h^2}(E - V_0) \text{ ----- (5)}$$

Substitute equations (4) and (5) in equations (2) and (3) respectively

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \text{ ----- (5a)}$$

and

$$\frac{d^2\psi}{dx^2} + \beta^2\psi = 0 \text{ ----- (6a)}$$

According to Bloch's theorem the solutions are of the form

$$\psi_k(x) = U_k(x)e^{ikx} \text{ ----- (2)}$$

Applying the conditions that ψ should be single valued and analytical, one can arrive at the solutions of the wave equations.

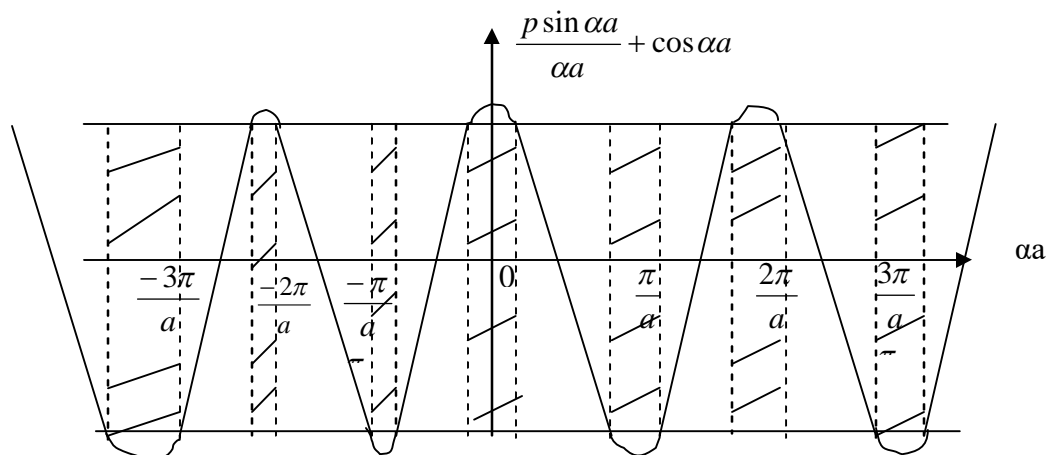
It is found that solutions of the wave equation are possible only for energies permitted by the equation.

$$P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka \text{ ----- (9)}$$

$$\text{Where } P = \frac{4\pi^2 ma}{h^2} V_0 b.$$

Interpretation of the conditions under which solutions are possible: - energy bands:

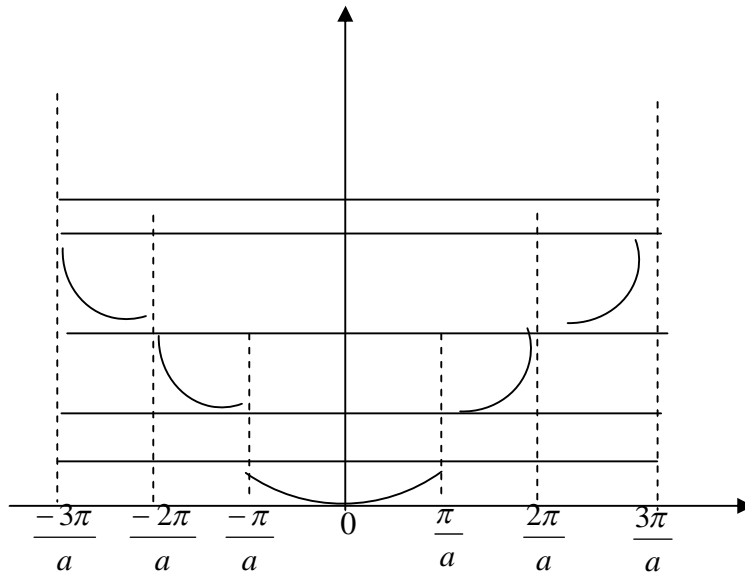
Consider the plot of the left hand side (LHS) of equation (9),



1. The right hand side (RHS) is a cosine term. It can vary only between the limits +1 and -1. As such the LHS can vary only between these limits. This means that ' αa ' can take only certain range of values. Such values of αa represent the allowed solutions. All other values of αa are forbidden.
2. The consequence of the conditions mentioned above can be understood with the help of above fig. It represents the LHS of equation (9) as a function (αa) the acceptable range of values of the LHS lies between the horizontal lines. Since α^2 is proportional to the energy E (eqn7) the abscissa will be a measure of the energy.
3. Since certain values of αa only are permitted. Energy is restricted to within certain ranges.
4. The energy spectrum of the electron consists of alternate regions of allowed (Shaded) and forbidden energy regions. These regions are conventionally referred to as allowed and forbidden energy bands.
5. The width of the allowed energy bands increases as the value of αa increases.

E-K relations:- The relation between energy and wave number for an electron in one-dimensional lattice can be understood via eqn(9) $[P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka]$ RHS of above

equation becomes ± 1 for values of $k = \pm \frac{n\pi}{a}$. These values actually define the boundaries of the allowed zones.



Discontinuities in 'E vs k' curve occur at $k = \pm \frac{n\pi}{a}$. The dotted curve indicates the parabola which a

free electron would exhibit $\left[E = \frac{h^2 K^2}{8\pi^2 m} \right]$.

Allowed energy values lie in the region $K = -\pi/a$ to π/a this zone is called the first Brillouin zone after a break in the energy values called forbidden energy band we have another allowed zone spread from $K = -\pi/a$ to $-2\pi/a$ and $+\pi/a$ to $2\pi/a$ this zone is called the second Brillouin zone. Similarly higher order Brillouin zones are formed.

UNIT-V
Assignment-Cum-Tutorial Questions

Section A: Objective Questions

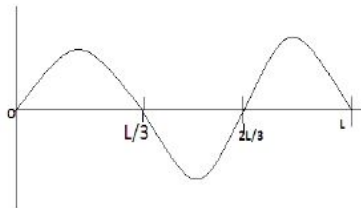
1. The expression for conductivity in a metal is given by _____.
2. The limits for first Brillouin zone are _____.
3. The expression for Bloch function is _____.
4. According to band theory, a freely moving electron inside a metal Experiences _____ potentials of positive ions. []
a) Constant b) periodic c) uniform d) consistent
5. According to Kronig penny model a metal contains _____ []
a) Potential wells b) Potential Barrier
c) Potential wells & Barriers d) None of the above
6. The Bloch function is a product of a plane wave function and a ----- []
a) Periodic function of the lattice with a period of inter- atomic distance
b) Non-periodic function with a period of inter- atomic distance
c) Periodic function with a period of half inter- atomic distance
d) None of the above
7. The metals possess high thermal and electrical conductivities because of []
(a) free electron movement in lattice (b) charged particles
(c) interaction between electrons and lattice ions
(d) repulsion between electrons
8. The Fermi surface need not always be: []
(a) spherical (b) cubic (c) parallelepiped (d) none
9. At absolute zero of temperature, the highest filled energy level of a metal is called []
(a) Fermi energy level (b) de Broglie energy level (c) Maxwell energy level (d) none
10. According to Pauli's exclusion principle, an energy level can accommodate not more than _____ electrons. []
(a) one (b) two (c) three (d) four
11. Relaxation time may be defined as the time taken by an electron to reduce its velocity to _____ of its initial value.
(a) half (b) one-third (c) (1/e) (d) 1/2e
12. Classical free electron theory of metals was introduced by: []
(a) P.Drude (b) G.P.Thomson (c) Albert Einstein (d) Newton
13. Quantum free electron theory of metals successfully explains: []
(a) electrical conductivity (b) specific heat and thermionic emission
(c) paramagnetism (d) all
14. The observed mean free paths of electrons in a metal are of []
(a) few Angstroms (b) ten Angstroms (c) several hundred Angstroms (d) none
15. At non-zero temperatures, the probability of filling Fermi energy level of a metal with electrons is []
(a) 1/3 (b) 2/3 (c) 1/2 (d) '0'

Section B: (Subjective Questions)

- 1) What are the assumptions of classical free electron theory? Obtain equation for conductivity basing on classical free electron theory.
- 2) Discuss the Kronig Penny model for the motion of an electron in a periodic potential.
- 3) What is Fermi Dirac distribution function? What is the significance of Fermi Dirac distribution function?
- 4) Show that when an electron travels in periodic potential provided by the lattice, it experiences allowed energy bands separated by forbidden regions.
- 5) Distinguish between classical free electron theory and quantum free electron theory of metals.
- 6) Explain briefly the quantum free electron theory of metals.
- 7) Explain the terms (i) mean free path, (ii) relaxation time and (iii) drift velocity of an electron in a metal.

Section C: (Additional Questions)

1. Find the ground state energy of an electron confined to a cubical box of side 1.2\AA . (Planck constant $h = 6.626 \times 10^{-34}$ J-s, Mass of the electron, $m_e = 9.1 \times 10^{-31}$ kg)
2. A free particle of mass m is confined to a region of length L . The de-Broglie wave associated with the particle is sinusoidal in nature as shown in figure. The energy of the particle is



- a) $9h^2 / 8mL^2$ b) $h^2/8mL^2$ c) $3h^2/8mL$ d) None of those

3. At what temperature we can expect a 10% probability the electrons in Silver have an energy which is 1% above the Fermi energy? ($E_F = 5.5$ eV for silver) (Take approximate value for answer)
(a) 170K (b) 250K (c) 290K (d) 310 K
4. Evaluate the Fermi function for energy $2kT$ above the Fermi energy.

Solid State Physics

Unit-VI

Magnetic materials

Course objectives

- To understand the origin of magnetism in materials

Syllabus:

Permeability, magnetization - Origin of magnetism - Classification of magnetic materials - Domain theory (qualitative) – Hysteresis - Soft and hard magnetic materials - Applications

Course Outcomes

Student will be able to

- apply the basic principles of magnetism exhibited by materials.
- derive orbital and spin contribution for magnetism

MAGNETIC MATERIALS

PERMEABILITY

Consider an unmagnetized bar of a magnetic material in a uniform magnetic field. It has been observed that the bar gets magnetized by induction and gets a polarity. After magnetization, the magnetic lines in the bar emanate from N-pole pass through the outer region and then re enter the S-pole. These lines form a closed loop within the magnet by passing from S-pole to N-pole.

- It will be interesting to know that the lines of the magnetized bar oppose the lines of the original field outside the magnet and favour inside the magnet.
- As a result of this the magnetic field strength (H) is increased inside the bar and decreased outside it. Similarly the magnetic flux density (B) becomes high inside the bar and low outside.
- We find that flux density (B) is directly proportional to the magnetic field strength (H). Mathematically $B \propto H$, $B = \mu H$ where μ is a constant of proportionality and is known as absolute permeability of the medium.
- If the flux density is established in air or vacuum or in a non-magnetic material then the above equation may be written as $B_0 = \mu_0 H$ where B_0 is the flux density in air or vacuum and μ_0 is the absolute permeability of air or vacuum.
- The ratio μ / μ_0 is known as the relative permeability of the medium. It is designed by the symbol μ_r . Mathematically $\mu_r = (\mu / \mu_0) = (B/B_0)$ it may be noted that the relative permeability of air or a non-magnetic material is unity.

Magnetization

The term magnetization may be defined as the process of converting non – magnetic bar into a magnetic bar.

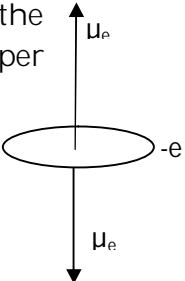
- The term is almost analogous to the polarization in dielectric materials. The flux density $B = \mu H = \mu_0 \mu_r H$, $B = \mu_0 \mu_r H + \mu_0 H - \mu_0 H = \mu_0 H + \mu_0 H(\mu_r - 1) = \mu_0 H + \mu_0 M$, $B = \mu_0 (H + M)$ where the magnetization, $M = H(\mu_r - 1)$. It is expressed in ampere/meter. From the equation, we find that if a magnetic field is applied to a material, the magnetic flux density is equal to the sum of the effect on vacuum and that on the material. The magnetization may thus be defined as the magnetic dipole moment per unit volume.

Origin of magnetic moment –Bohr Magnetron

Electric current in atoms:

For simplicity, let us consider the simplest atom of hydrogen in which one electron revolves round the proton say on a circular path of radius r.

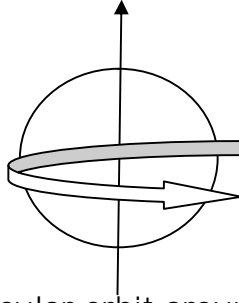
- At any instant the electron at appoint P and the proton at the centre form an electric dipole, the direction of the dipole goes on changing as the electron moves further. Obviously the time average of the electric dipole moment is zero and there will be no electro static field at a distance point due to this dipole.
- Again the moving electron is equivalent to a current ring. If v is the velocity of the electron it will make $V / 2\pi r = w / 2\pi$ revolutions per second and it is equivalent to a current given by $I = ew / 2\pi$
- The circular path is equivalent to a magnetic dipole and the magnetic moment due to this orbital motion of the election is.



$$\mu_{el} = IA = (ew / 2\pi)\pi r^2 = \frac{ewr^2}{2}$$

- The angular momentum associated with the electronic motion is given by mr^2w .
- Hence we can relate the magnetic dipole moment and angular momentum as $\mu_{el} = -e / 2m \times$ angular momentum.
- The minus sign indicates that the dipole moment points in a direction opposite to the vector representing the angular momentum. A substance therefore possesses permanent magnetic dipoles if the electrons of its constituent atom have a net non vanishing angular momentum.
- The ratio of the magnetic dipole moment of the electron due to its orbital motion and the angular moment of the orbital motion is called orbital gyro magnetic ratio, or orbital magneto mechanical ratio of an electron, represented by γ .
- In our case it is given by $\gamma =$ magnetic moment / Angular momentum = $e / 2m$. According to modern atomic theory the angular momentum of an electron in the orbit is determined by the orbital quantum number l which is restricted to set of values $l = 0, 1, 2, \dots, (n-1)$ where n is the principal quantum number which determines the energy of the orbit. It can accept only the integer values $n = 1, 2, 3, 4, \dots$. The corresponding electronic shells are called K, L, M, N, \dots shells.
- The angular momentum of electrons associated with a particular value of l is given by $l(h / 2\pi)$. The electrons associated with the states $l = 0, 1, 2, 3, \dots$ are called S, P, d, f, \dots electrons respectively.
- The strength at the permanent magnetic dipole is given by $\mu_{el} = -(e / 2m)(lh / 2\pi) \rightarrow \mu_{el} = -ehl / 4\pi m = -\mu_b \cdot l$. The quantity $\mu_b = eh / 4\pi m$ is an atomic unit called Bohr magneton and has a value 9.27×10^{-24} ampere metre² and this represents the magnetic moment of an elementary permanent magnetic dipole.
- From the above equation it is clear that electron in an atom can take only certain specified values of magnetic moment depending on the value of l . Hence for s electron $l = 0$ and magnetic moment is zero.

ELECTRON SPIN AND MAGNETIC MOMENT:



Besides rotating on a circular orbit around the positive nucleus, an electron also rotates around an axis of its own.

- The magnetic moment associated with spinning of the electron is called spin magnetic moment μ_{es} .
- Although, strictly speaking it has a quantum-mechanical origin. We can think of its magnetic moment as being due to the rotation of the electronic charge about one of the diameters of the electron in a manner similar to that of the earth's spinning motion around its north south axis.
- If we consider the simple case of an electronic charge being spread over a spherical volume the electron spin would cause different charge elements of this sphere to form closed currents resulting thereby in a net spin magnetic moment.
- Clearly this net magnetic moment would depend upon the detailed structure of the electron and its charge distribution.
- It turns out that an equation connects μ_{es} with spin angular momentum S by the relation $\mu_{es} = \gamma(e/2m)s$ where the value of the coefficient γ called the spin gyro magnetic ratio depends on the structure of the spinning particle and its charge distribution. The experimental value of γ for an electron is (-2.0024) the negative sign indicating the μ_{es} in a direction opposite to that of $s = h/4\pi$ for an electron.

$$\mu_{es} = \frac{-eh}{4\pi m} = 9.27 \times 10^{-24} \text{ ampere - metre}^2$$

Thus the magnitude of magnetic moments due to spin and the orbital motions of an electron are of the same order of magnitude. It may be mentioned that spin and μ_{es} are intrinsic properties of an electron and exist even for a stationary electron ($L=0$).

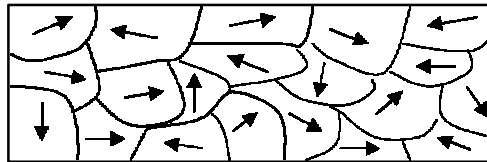
Classification of magnetic materials

Type	χ	Temperature dependence of χ	Effects of H on χ	Magnetic flux B	Example
Diamagnetic	Small, <0	Independent	Independent	$B_{out} \gg B_{in}$	Organic solids like benzene, naphthalene
Paramagnetic	Small, +ive	$\chi = C/T$	Decreases with H	$B_{in} > B_{out}$	Alkali metals, Rare earth
Ferromagnetic	Very large +ive	$\chi = C/T - \theta$	Increases initially and then decreases	$B_{in} \gg B_{out}$	Transition metals

DOMAIN THEORY (QUALITATIVE)

Ferromagnetic domain structure:

According to the Weiss molecular field theory, the ferromagnetic material consists of small regions called domains as shown in fig.1.



Polycrystalline

Fig1.domain structure of ferromagnetic material

- A domain is a small region in a ferromagnetic material in which all magnetic spin dipoles are aligned parallel to one another in the same direction even in the absence of external magnetic field i.e., it is spontaneously magnetized in the absence of magnetic field as shown in fig.2.
- This magnetization depends on temperature. These domains are having dimensions of $\sim 10^{-2}$ cm and it contains as many as 10^{17} atoms. Due to the nature of the domain structure, in its pure state it is un-magnetized as shown in fig.1.

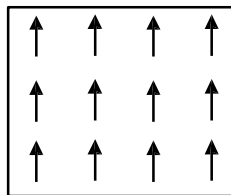


Fig.2. A single domain in a ferromagnetic material- spontaneous magnetization.

- In the presence of an external magnetic field, the ferromagnetic material acquires a non-zero magnetization.
- This may occur in two different ways: by increase in the volume of the domains which are favorably oriented with respect to the field at the expense of unfavorably oriented domains at weak magnetic field (Domain growth in a weak external magnetic field) as shown in fig.3. (b),
- By the rotation of the domain magnetization towards the external

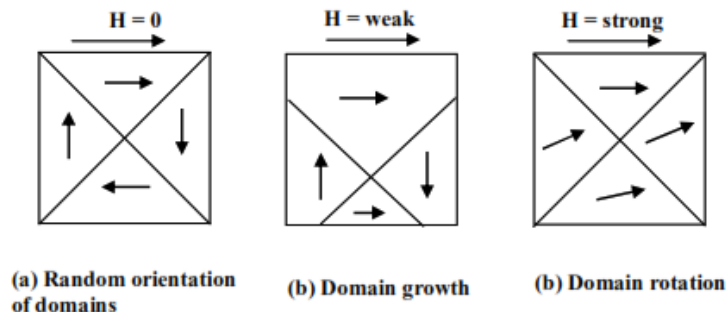


Fig.3. Domain structure of ferromagnetic material (a) random orientation in the absence of magnetic field (b) domain growth in the weak magnetic field (c) domain rotation in the strong magnetic field.

field direction at strong magnetic field (Domains rotation in a strong external magnetic field) as shown in fig 3.(c).

Domain growth in a weak external magnetic field:

When the magnetic field is applied to the ferromagnetic material in some arbitrary direction, those domains which are favorably oriented along the magnetic field direction will grow at the expense of the unfavorably oriented domains, hence the volume of the favorably oriented domains increases i.e., domain growth occurs.

- This is due to the motion of domain walls as shown in the fig.3. (b) . As a result, the more number of magnetic spin dipoles enter into the favorable domains from the un-favorably oriented domains and hence the magnetization increases. In pure materials, due to this process, the magnetization is reversible in the weak external field.

Domains rotation in a strong external magnetic field :

As the external magnetic field is further increased, the magnetization gradually increases and reaches a saturation value at a particular magnetic field. It occurs due to sudden rotation of un-favorably oriented domains along the magnetic field direction. In a strong external magnetic field, the domain magnetization direction is rotated in the field direction as given in fig.3. (c) Thus in strong magnetic field, the magnetization in the domains increases with irreversible wall motion. The substance is said to be saturated and it gets maximum magnetization.

(e). Hysteresis effect in ferromagnetic material:

Hysteresis is a well known property of ferromagnetic materials as shown in fig.4. Hysteresis is a lack of re-traceability of magnetization curve of a

ferromagnetic material during magnetization and demagnetization processes.

- When a ferromagnetic material is magnetized completely in one direction, it will not relax back to zero magnetization when the imposed magnetizing field is removed. It is related to the existence of magnetic domains in ferromagnetic material and provides to understand of magnetic properties of materials.

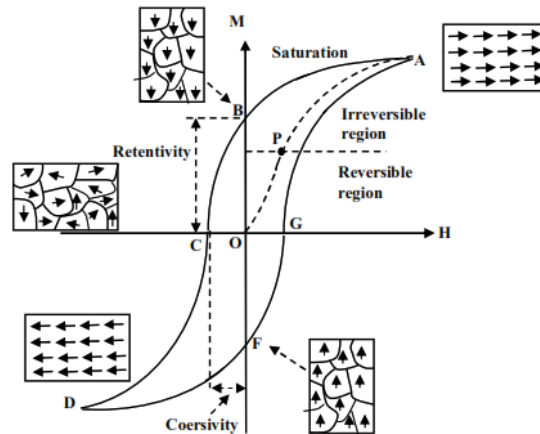


Fig.4. The hysteresis loop

Hysteresis effect in ferromagnetic materials can be understood by considering the growth of domains and rotation of domains and it has the following properties: Saturation magnetization, Remanent magnetization, Coercive field and Hysteresis loss. Based on these properties the ferromagnetic materials are two types: soft and hard ferromagnetic materials which have many practical applications.

HYSTERESIS CURVE:-

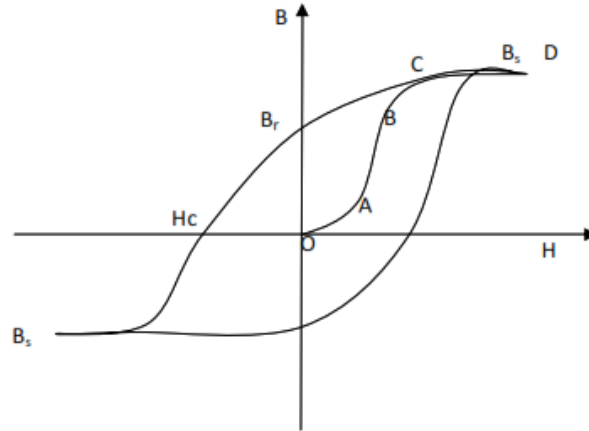
When a ferromagnetic or a ferromagnetic specimen is subjected to a magnetic field which varies cyclically the magnetization of the specimen which is realized in terms of the magnetic induction B , fails to keep in step with the applied field. This property and its consequences are studied here.

Hysteresis is the sluggishness or the phase lag of the magnetic induction B in ferromagnetic materials. With respect to the cyclic variation of an applied magnetic field, when the specimen is at a temperature below its curie temperature.

- To study the dependence of B on H , a ferromagnetic specimen is subjected to a magnetic field varying to only in magnitude but also in direction. The strength of the magnetic field H is increased from a zero value to a high value and then reduced to zero value; again it is increased to a high value but in the opposite direction and brought back to zero value and then increased again by reversing its direction. As the magnetic field is varied a corresponding variation in the magnetic induction B of the specimen is

observed. But the notable feature is that the two variations are not synchronized B trails behind H

- The value of B also increases and the curve will develop along OD. After the point D, B remains constant in spite of continued increase in H. This value of B at D is called the saturation value and the state of specimen is referred to as saturation magnetization. Now when the value of H is decreed B starts decreasing from the point D,



When H becomes zero, the value of B retains a value equal to B_r . This residual value of B equal to OB_r is called remanent magnetization or retentive magnetization. The value of B goes to zero and the specimen gets completely demagnetized only when the direction of H is reversed and attains a value OH_c . This value of $H = OH_c$ is called coercive force and the effect is called coercivity. Further increase in H causes the specimen to get magnetized in the opposite direction which means B also increases in the opposite direction and reaches saturation value. The variation of B with respect to H traced along the closed path for a ferromagnetic specimen refers to one full cycle of magnetization and demagnetization and is called the Hysteresis loop or the Hysteresis curve.

SOFT AND HARD MAGNETIC MATERIALS:

Ferromagnetic materials are classified as soft or hard. The classification is mainly based on the hysteresis characteristics magnetically soft materials have low coercivity (H_c) while those with high values of H_c are called hard magnetic materials.

Hard

1. Large hysteresis loop area – high energy loss High remanent magnetization
2. High Coercivity
3. Low permeability
4. Low susceptibility

Soft

1. Low remanent maganetization
Low Hysteresis loss
2. Low coercivity
3. High permeability
4. High susceptibility

Hard magnetic materials are used to manufacture permanent magnets low magnetic materials are used in A.C. fields.

UNIT-VI

Assignment –cum- tutorial questions

Section A: Objective Questions

1. The magnetic moment of a free atom is due to _____
2. Susceptibility of diamagnetic materials is _____
3. In diamagnetic materials, the net magnetic moment of all the atoms is _____
4. The temperature independent susceptibility is _____
5. Diamagnetism is exhibited by all materials. (T/F)
6. Choose the correct option
 - a) $\chi < 1$ for dia, $\chi > 1$ for para, $\chi \ll 1$ for ferromagnets
 - b) $\chi < 1$ for dia, $\chi > 1$ for para, $\chi \gg 1$ for ferromagnets
 - c) $\chi < 1$ for dia, $\chi \gg 1$ for para, $\chi \ll 1$ for ferromagnets
 - d) $\chi < 1$ for dia, $\chi \ll 1$ for para, $\chi \gg 1$ for ferromagnets
7. Hysteresis curve or B-H curve is variation of
 - a) Magnetic induction with magnetic field intensity
 - b) Magnetic field with dipole moment
 - c) Magnetization with susceptibility
 - d) Susceptibility with dipole moment
8. When an external field is applied to a magnetic material,
 - a) A torque acts on individual dipole moments.
 - b) No force is experienced by the dipole moments.
 - c) The particles are in random motion
 - d) There is no change in the dipole moments of molecules.
9. Magnetic field strength in silicon is 1000 Am^{-1} . If the magnetic susceptibility is 3×10^{-7} , calculate the magnetization.
10. A magnetic material has a magnetization of 3000 A/m and flux density of $5 \times 10^{-3} \text{ Wbm}^{-2}$. The relative permeability of the material is _____
11. Stationary charged particles exhibit magnetism. (T/F)
12. Materials do not having permanent magnetic dipoles are
 - a) Paramagnetic
 - b) ferromagnetic
 - c) ferromagnetic
 - d) diamagnetic
13. When a diamagnetic material is placed inside an external magnetic field
 - a) Induced magnetic dipoles act along the applied field direction .
 - b) Induced magnetic dipoles act opposite to the applied field direction.
 - c) Induced magnetic dipoles act perpendicular to the applied.
14. Diamagnetic susceptibility is
 - a) Large, negative
 - b) small, positive
 - c) small, negative
 - d) large, positive
15. Which of the following statement is wrong?
 - a) Diamagnetic property is independent of temperature
 - b) Paramagnetic property is dependent of temperature
 - c) Paramagnetic property is independent of temperature
 - d) Ferromagnetic property is dependent of temperature
16. Magnetic susceptibility X is equal to
 - a) The ratio of the magnetic induction in the sample to the applied magnetic field intensity.
 - b) Magnetic moment per unit volume

- c) The ratio of the intensity of magnetization produced in the sample to the magnetic field intensity producing it.
- d) The ratio of the magnetic field intensity to the intensity of magnetization.
17. Relative permeability is related to magnetic susceptibility by
- a) $\mu_r = 1 - \chi$ b) $\mu_r = \chi - 1$ c) $\mu_r = 1 + \chi$ d) $\mu_r = 1/\chi$
18. Choose the wrong statement
- a) Materials lacking permanent dipoles are called diamagnetic
- b) Materials possessing permanent dipoles which do not interact among themselves are called paramagnetic
- c) Materials possessing permanent dipoles which strongly interact among themselves are called ferromagnetic
- d) Materials in which magnitudes of permanent dipoles aligned antiparallel is equal are called antiferromagnetic.
19. The unit of magnetic field intensity is
- a) $A\,m^{-1}$ b) $H\,m^{-1}$ c) $Wb\,m^{-2}$ d) no units
20. Magnetic induction B is related to magnetic field intensity H by the relation
- a) $B = \mu_0(H - m)$ b) $B = \mu_0 \frac{H}{M}$ c) $B = \mu_0 \frac{M}{H}$ d) $B = \mu_0(H + M)$
21. One Bohr magneton equals
- a) $9.27 \times 10^{-24} \text{ A}\cdot\text{m}^2$ b) $9.1 \times 10^{-31} \text{ A}\cdot\text{m}^2$
- c) $9.27 \times 10^{-16} \text{ A}\cdot\text{m}^2$ d) $9.1 \times 10^{-24} \text{ A}\cdot\text{m}^2$
22. The temperature at which the transition of antiferro to para magnetism take place is called.
- a) Curie – Weiss temp. b) Curie temperature
- c) Debye temp. d) Neel temp.
23. The transition from the ferromagnetic to the paramagnetic state is named after
- a) Curie b) Curie-Weiss c) Neel d) Debye
24. Orbital magnetic dipole moment μ_0 of an revolving electrons is given by
- a) $\mu_0 = \left(\frac{er^2w_0}{2} \right)$ b) $-\frac{e^2r}{2w_0}$ c) $-\left(\frac{er^2w_0}{2} \right)$ d) $\frac{e^2r}{2w_0}$

Section B: (Subjective Questions)

Long answer questions

1. What is the origin of magnetism? Derive the expression for magnetic moment due to orbital motion and spin motion of charged particles.
2. Distinguish between dia, para and ferromagnetic materials.
3. How can the hysteresis curve be accounted based on domain theory?
4. Explain B-H curve for ferromagnetic materials.

Short answer questions

1. What are the sources of permanent dipole moment in magnetic materials?
2. Define one Bohr magneton.
3. What is magnetic susceptibility?
4. Draw the variation of susceptibility with temperature for dia, para and ferromagnetic materials.
5. What do you mean by hysteresis?
6. What do you mean by hysteresis losses?

Section C: (Additional Questions)

1. The electron in a hydrogen atom moves in a circular orbit of radius 0.5 \AA . The electron performs 700 revolutions per second. Determine the magnetic moment.
2. The magnetizing field strength (H) in a piece of ferric oxide is 10^6 A/m . If the susceptibility (χ) of the material at room temperature, $T = 300 \text{ K}$ is 1.0×10^{-3} , calculate the magnetic flux density (B) in the material.